MTP-AERO-63-38
May 20, 1963

GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

A SURVEY OF THE INFLUENCE OF VARIATIONS
IN STAGE CHARACTERISTICS ON OPTIMIZED TRAJECTORY SHAPING

PART II: TRANSFER FROM CIRCULAR ORBITS
INTO A SPACE FIXED ELLIPSE

By

Gary McDaniel

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ABSTRACT

Trajectories have been investigated for upper stage vehicles, characterized by thrust to weight ratio and specific impulse, departing from a circular orbit and injecting into an elliptical orbit. The orbits are co-planar and both cases where the initial orbit intersects or does not intersect the terminal orbit have been considered.

Two families of solutions have been found for a given thrust to weight ratio and specific impulse. Emphasis has been directed toward weight into orbit, trajectory shapes, tilt angle histories, and angle of attack histories for both families.

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FUTURE PROJECTS BRANCH
AEROBALLISTICS DIVISION

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SUMMARY

Trajectories have been investigated for upper stage
vehicles, characterized by thrust to weight ratio and
specific impulse, departing from a circular orbit and
injecting into an elliptical orbit. The orbits are
coplanar and both cases where the initial orbit inter-
sects or does not intersect the terminal orbit have
been considered.

Two families of solutions have been found for a given
thrust to weight ratio and specific impulse. Emphasis has
been directed toward weight into orbit, trajectory shapes,
tilt angle histories, and angle of attack histories for
both families.

SECTION I. PROBLEM DESCRIPTION AND APPROACH

This study is concerned with trajectory shapes, weight
into orbit, and control histories of an upper stage vehicle
departing from circular orbits and injecting into a space-
fixed elliptical orbit. Different types of vehicles are
considered and are characterized by their thrust to weight
ratio at ignition, F/W_o, and specific impulse, I_{sp}. These
two parameters have been restricted to $.5 \leq F/W_o \leq 2$ and
$400 \leq I_{sp} \leq 1500$.

The initial circular orbits considered are coplanar
with the terminal elliptical orbit. Three initial circular
orbits have been considered with altitudes, y_o, of 100,
200 and 300 km. The elliptical orbit has an altitude at
perigee of 150 km. Hence, the 200 and 300 km orbits
intersect the terminal orbit.

2

The two quantities, energy and angular momentum, which define the shape of the ellipse were taken from a trajectory which circumnavigates the moon and re-enters into the gravitational field of the earth. This particular ellipse has energy and angular momentum values per unit mass of -1.495811×10^6 (m^2/sec^2) and 7.1158378×10^{10} (m^2/sec), respectively. These values yield a total flight time of approximately 122 hours. This particular elliptical orbit was chosen in order that the results in this study may be of significance in future lunar studies.

The motion is assumed to take place in two dimensions in the gravitational field of a spherical earth free of atmosphere. When the initial conditions have been specified along with a F/W_o and I_{sp} value the vehicle's trajectory is numerically integrated in a $(r, v, \$)$ coordinate system. The burning time of the trajectory is extremized by the calculus of variations technique. Since this is a problem of optimum orbital transfer between a circular orbit and an elliptical orbit which is space-fixed, a phi-independent program was used, where phi is the angle between the launch vertical and the local vertical on the trajectory. This implies that ignition can occur at any time on the initial orbit and the line of apsides of the terminal orbit is fixed.

In order that the end conditions are achieved the initial angle of attack and its time derivative, α_o and $\dot{\alpha}_o$, have to be determined. This is a numerical procedure that determines α_o and $\dot{\alpha}_o$ by an interpolation or extrapolation on several cases which approach the desired end conditions.

α_o and $\dot{\alpha}_o$ have been chosen as the isolation parameters rather than the Lagrangian multipliers, λ_i's. This transformation is easily seen. Consider the differential equations of constraint

$$\dot{r} - v \cos \$ = G_1$$

$$\dot{v} - \frac{F}{m} \cos \alpha + g \cos \$ = G_2 \qquad (1)$$

$$\dot{\$} - \frac{F}{mv} \sin \alpha - \left(\frac{g}{v} - \frac{v}{r}\right) \sin \$ = G_3$$

along with the function to be extremized

$$f = 1 + \sum_{i=1}^{n} \lambda_i \, G_i \, , \qquad i = 1, \, \dots, \, n. \tag{2}$$

The Euler-Lagrange equation where α is the independent variable is

$$\frac{\partial f}{\partial \alpha} - \frac{d}{dt} \left(\frac{\partial f}{\partial \dot{\alpha}} \right) = 0. \tag{3}$$

By applying (3) to (1) and (2) one obtains

$$\lambda_2 \, \frac{F}{m} \, \sin \alpha - \lambda_3 \, \frac{F}{mv} \, \cos \alpha = 0 \qquad \text{or} \qquad .$$

$$\lambda_2 \, v \, \sin \alpha = \lambda_3 \, \cos \alpha. \tag{4}$$

Using the trigonometric identity $\sin^2 \alpha + \cos^2 \alpha = 1$ (4) becomes

$$\cos \alpha = \frac{\lambda_2 \, v}{\sqrt{\lambda_3^2 + \lambda_2^2 \, v^2}} \tag{5}$$

$$\sin \alpha = \frac{\lambda_3}{\sqrt{\lambda_3^2 + \lambda_2^2 \, v^2}}$$

By differentiating (5) with respect to time the equation for $\dot{\alpha}$ is determined.

SECTION II. DISCUSSION

When F/W_O, I_{sp}, and y_O are specified along with α_O and $\dot{\alpha}_O$, a trajectory is determined. In this study α_O has been parameterized in order that all permissible trajectories can be investigated.

For a given F/W_O, I_{sp}, y_O, and α_O a trajectory is computed until the energy condition has been attained. $\dot{\alpha}_O$ is then varied until the angular momentum condition has been satisfied along with the energy condition.

It has been found that for some interval on α_O, $\alpha_{OL} < \alpha_O < \alpha_{OR}$, there is a pair of $\dot{\alpha}_O$'s for each α_O in this interval which satisfies the end conditions. This can be seen in Figures (1 - 6) where trajectories have been computed for different $\dot{\alpha}_O$'s when only the energy condition has been achieved. These figures were obtained using a $F/W_O = .5$ and one with an $I_{sp} = 700$ sec for $y_O = 100$, 200, and 300 km. Other diagrams similar to Figures (1 - 6) will not be shown since different parameters produce a similar effect.

The angular momentum taken from these trajectories has been plotted as a function of $\dot{\alpha}_O$ for parameters of α_O. In this interval on $\dot{\alpha}_O$ the angular momentum attains a maximum, or the change of angular momentum with respect to $\dot{\alpha}_O$, $\dfrac{\Delta M}{\Delta \dot{\alpha}_O}$, goes to zero.

For convenience this maximum value for the angular momentum will be expressed as M_{max} and the angular momentum of the desired ellipse will be expressed as M^*. If $M_{max} > M^*$ for some α_O then $\alpha_{OL} < \alpha_O < \alpha_{OR}$ and there are two solutions corresponding to

$$\frac{\Delta M}{\Delta \dot{\alpha}_O} > 0 \text{ or } \frac{\Delta M}{\Delta \dot{\alpha}_O} < 0$$

evaluated at M^*. If $M_{max} = M^*$ then there is only one solution. This defines the lower and upper limits on α_O, α_{OL} and α_{OR}. If $M_{max} < M^*$ then $\alpha_O < \alpha_{OL}$ or $\alpha_{OR} < \alpha_O$ and the desired end conditions can not be satisfied.

This result has been consistent with all parameters F/W_O, I_{sp}, and y_O. This, of course, is not inconsistent with the calculus of variations formulation since it guarantees only a relative maximum (minimum) in the two point boundary value problem with variable end points.

SECTION III. RESULTS

Trajectories were computed for parameters of α_O where $\alpha_{O_L} < \alpha_O < \alpha_{O_R}$ for both solutions corresponding to

$$\frac{\Delta M}{\Delta \dot{\alpha}_O} > 0 \quad \text{and} \quad \frac{\Delta M}{\Delta \dot{\alpha}_O} < 0.$$

Hereafter, a T_1 solution will refer to the $\frac{\Delta M}{\Delta \dot{\alpha}_O} > 0$ solution

and a T_2 solution will refer to the $\frac{\Delta M}{\Delta \dot{\alpha}_O} < 0$ solution.

From these trajectories we have investigated weight into orbit, trajectory shapes, and control histories.

Figures (7 - 12) illustrate the ratio of the weight into orbit to the initial weight for different parameters of F/W_O, I_{sp}, and y_O as a function of α_O. This ratio is is designated as $\frac{W_c}{W_O}$. Subscripts of L and R on this ratio will refer to the T_1 and T_2 solution, respectively.

When α_O was chosen near α_{O_L} and was allowed to increase in the positive direction the T_1 solutions yielded $\frac{W_c}{W_O}$ values greater than the T_2 solutions. However, as α_O increased the difference between $\frac{W_c}{W_O}$ for the two solutions decreased until for some α_O^*

$$\left(\frac{W_c}{W_O}\right)_L - \left(\frac{W_c}{W_O}\right)_R = 0. \quad \text{In the interval } \alpha_{O_L} < \alpha_O < \alpha_O^*, \left(\frac{W_c}{W_O}\right)_L$$

attained a maximum value. This was consistent for all values of F/W_O, I_{sp}, and y_O considered. These maximum values of $\left(\frac{W_c}{W_O}\right)_L$

have been plotted as a function of y_O in Figures (13 - 15).

$(\frac{W_c}{W_O})_R$ did not always yield a maximum as a function of α_O. Certain restrictions were found on F/W_O, I_{sp} and y_O. These restrictions were:

1. For $y_O = 100$ km $(\frac{W_c}{W_O})_R$ did not maximize for any F/W_O and I_{sp} considered. Here, α_{O_R} appears to be equivalent to α_O^*.

2. For $y_O = 200$ and 300 km $(\frac{W_c}{W_O})_R$ did maximize for $F/W_O = 1$ and $F/W_O = 2$. This maximum occurs in the interval $\alpha_O^* < \alpha_O < \alpha_{O_R}$.

3. For $F/W_O = .5$ $(\frac{W_c}{W_O})_R$ did maximize for $y_O = 300$ km with $I_{sp} = 400$ and 500 sec. This maximum occurs in the interval $\alpha_O^* < \alpha_O < \alpha_{O_R}$.

Since $(\frac{W_c}{W_O})_R$ did not maximize for $y_O = 100$ km for the different F/W_O and I_{sp} values, graphs similar to Figures (13 - 15) can not be shown for the T_2 solutions.

For launch window considerations the angle ω is defined as the angle between the launch vertical and perigee of the terminal orbit. ω is negative when measured clockwise from perigee and positive when measured counter-clockwise. ω is determined once the end conditions have been satisfied. Remembering that the end conditions are determined by α_O and $\dot{\alpha}_O$ it is obvious that a non-optimum α_O yields a non-optimum ω, whereas, an optimum α_O yields an optimum ω. In this study an optimum α_O is defined as the α_O which maximizes $(\frac{W_c}{W_O})$.

Thus, a launch window representation can be given with $\frac{W_c}{W_O}$ as a function of $\Delta\omega$, where $\Delta\omega$ is defined as the deviation between non-optimum ω's and the optimum ω, i.e., $\Delta\omega = \omega_{n-opt} - \omega_{opt}$. If $\Delta\omega$ is negative this will imply

departure is prior to the optimum ω and if $\Delta\omega$ is positive this will imply departure is after the optimum ω.

$\dfrac{W_c}{W_o}$ has been plotted as a function of $\Delta\omega$ in Figures (16 - 28) for the different F/W_o, I_{sp}, and y_o values for both types of solutions. Figures (16 - 24) are the T_1 solutions and Figures (25 - 28) are the T_2 solutions. These graphs were obtained with α_o being in the interval $\alpha_{oL} < \alpha_o < \alpha_{oR}$. Unfortunately, we were not able to obtain $\alpha_o = \alpha_{oL}$ and α_{oR} which would imply the limits on ω, i.e., $\omega = \omega_L$ and $\omega = \omega_R$. However, we were able to obtain ω's near ω_L and ω_R. Due to this it should be emphasized that the reader of this report should not extrapolate values of $\dfrac{W_c}{W_o}$ as a function of $\Delta\omega$.

By comparing the T_1 solutions with the T_2 solutions on Figures (16 - 24) with Figures (25 - 28) a distinction is seen between the two sets of solutions. For the T_1 solutions it is seen that the largest portion of the interval on $\Delta\omega$ is on the positive axis, whereas, for the T_2 solutions the largest portion of $\Delta\omega$ is on the negative axis. In other words, there is more time alloted after the optimum ω for the T_1 solutions and more time alloted prior to the optimum ω for the T_2 solutions. The rate of change of ω with respect to time, $\dot{\omega}$, is shown in Figure (29) as a function of y_o in order that ω may be converted to time.

The reader can easily see the effects that the parameters F/W_o, I_{sp}, and y_o have on the size of a launch window.

The trajectory shapes for both types of solutions have been plotted (Figures 30 - 48). For better detail, altitude versus range has been plotted. This has been done for non-optimum and optimum situations. The trajectories shown in Figures (30 and 31) are typical examples for a non-optimum α_o. These results were obtained with a $F/W_o = .5$, $I_{sp} = 400$ and 900 sec, and $\alpha_o = -40°$. Figures (32 and 33) are non-optimum examples similar to Figures (30 and 31) except $F/W_o = 1$ and $\alpha_o = -20°$. The optimum trajectories are shown in Figures (34 - 48) for the different F/W_o and I_{sp} values.

From these illustrations it is seen that the T_2 solutions are initially away from perigee and inject into the terminal

orbit near perigee, whereas, the T_1 solutions are initially near perigee and injection is away from perigee. This result is typical for non-optimum and optimum trajectories.

The angle, χ, between the launch vertical and the vehicle's thrust vector has been plotted as a function of time in Figures (49 - 58). Non-optimum χ programs have been plotted in Figure (49) in order to illustrate the behavior of χ for α_0's in the interval $\alpha_{0L} < \alpha_0 < \alpha_{0R}$ for both solutions. This is typical for the different values of F/W_0, I_{sp}, and y_0. The parameters chosen in this non-optimum example are $F/W_0 = 1$, $I_{sp} = 700$ sec, and $y_0 = 300$ km.

Optimum χ programs are shown in Figures (50 - 58) in terms of the two solutions or only in terms of the T_1 solutions if the T_2 solutions do not apply. The first set of these Figures (50 - 52) are for the non-intersecting case, i.e., $y_0 = 100$ km. Notice in particular for the lower F/W_0, i.e., .5 in Figure (50) χ appears to be linear in time. In Figures (51 and 52) χ becomes non-linear with the extreme case being the higher $F/W_0 = 2$. Only the T_1 solutions apply to these figures.

In Figure (53) for $y_0 = 200$ km χ appears to be linear in time again for $F/W_0 = .5$. Again, only the T_1 solutions apply to this figure. In Figures (54 and 55) both solutions apply. Notice that the χ programs for both solutions appear linear and are essentially parallel.

Figures (56 - 58) illustrate χ for $y_0 = 300$ km for both types of solutions. Figure (56) shows χ is non-linear for $F/W_0 = .5$. In Figures (57 and 58) the two solutions appear to be linear and parallel.

The angle of attack, α, has been plotted as a function of time in Figures (59 - 68). α is the angle between the velocity vector and the thrust vector measured positive counter-clockwise. These α programs are the typical α-histories associated with the χ programs in Figures (49 - 58).

REFERENCE

1. Davidson, Mirt C., "A Survey of the Influence of Variations In Stage Characteristics on Optimized Trajectory Shaping, Part I: Two Stage Vehicle Injection Into Circular Orbit," MTP-AERO-63-29 dated April 24, 1963.

FIG. 1.   VARIATION OF TERMINAL ANGULAR MOMENTUM

WITH $\dot{\alpha}_o$ FOR PARAMETERS OF α_o

FOR $F/W_o = .5$; $y_o = 100$ km; $I_{sp} = 700$ sec

Angular Momentum $\times 10^{-10}$ (m²/sec)

Ang Mom of
Desired Ellipse

α_0 (deg)

7.4

7.3

7.2

7.0

6.9

6.8

-20

0
-40

35

20

30

40

-60

-.2   -.1   0   .1   .2   .3   .4

$\dot{\alpha}_0$ (deg/sec)
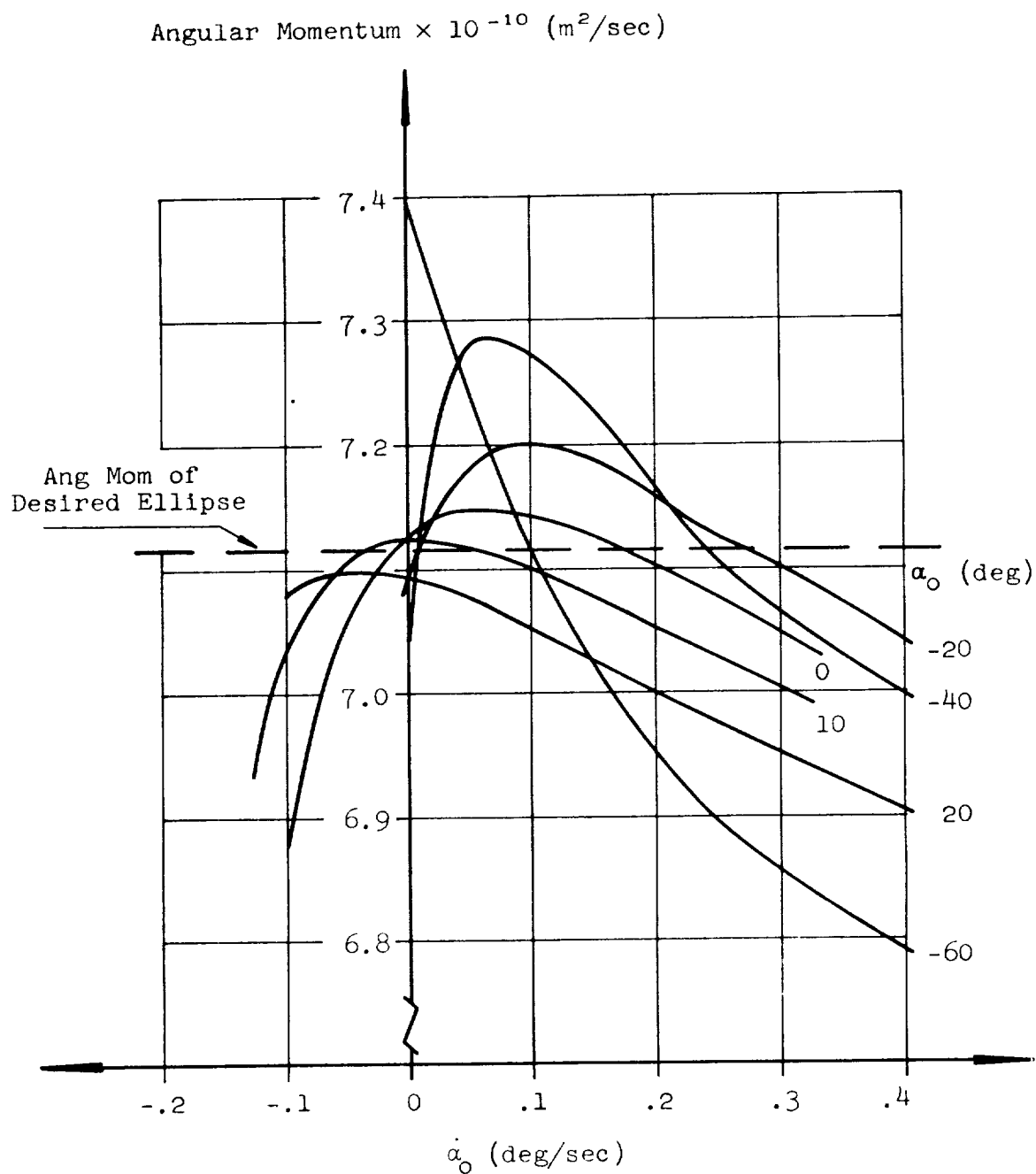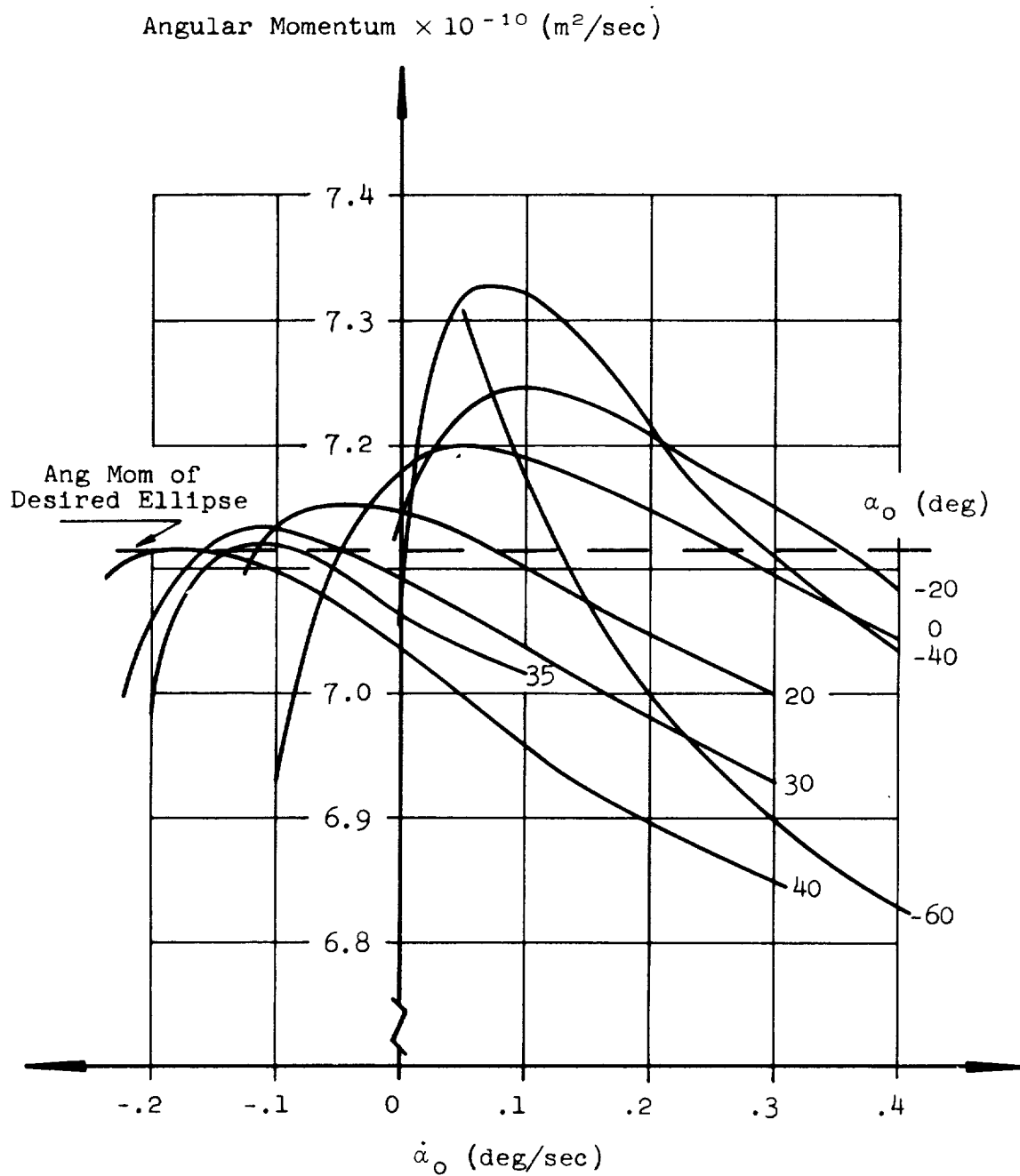
FIG. 2. VARIATION OF TERMINAL ANGULAR MOMENTUM

WITH α_0 FOR PARAMETERS OF α_0

FOR F/W$_0$ = .5; y_0 = 200 km; I_{sp} = 700 sec
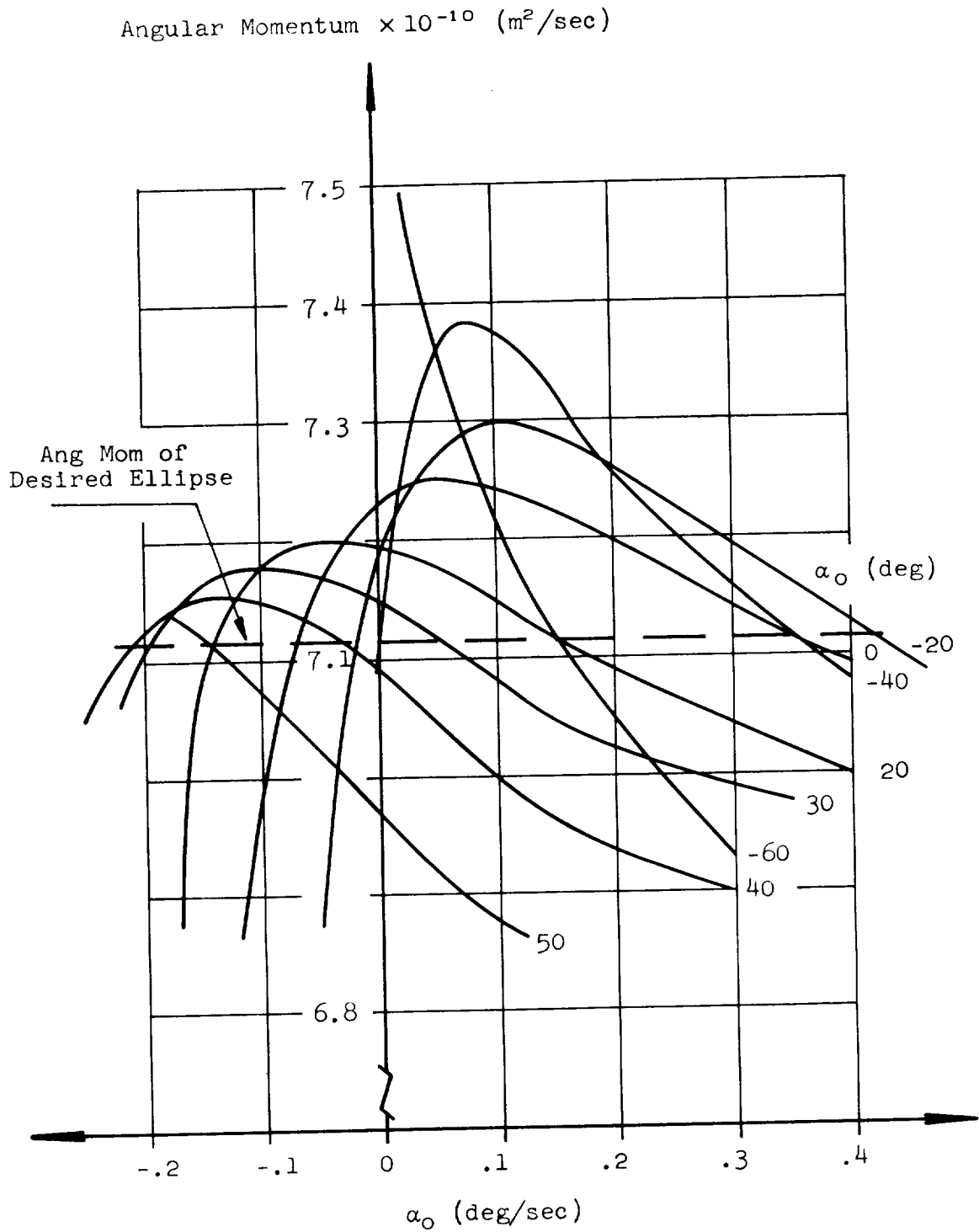
FIG. 3. VARIATION OF TERMINAL ANGULAR MOMENTUM

WITH $\dot{\alpha}_o$ FOR PARAMETERS OF α_o

FOR $F/W_o = .5$; $y_o = 300$ km; $I_{sp} = 700$ sec

Angular Momentum $\times$ 10^{-10} (m²/sec)

Ang Mom of
Desired Ellipse

α_o (deg)

-10

-20
-40

0

-60

20

$\dot{\alpha}_o$ (deg/sec)
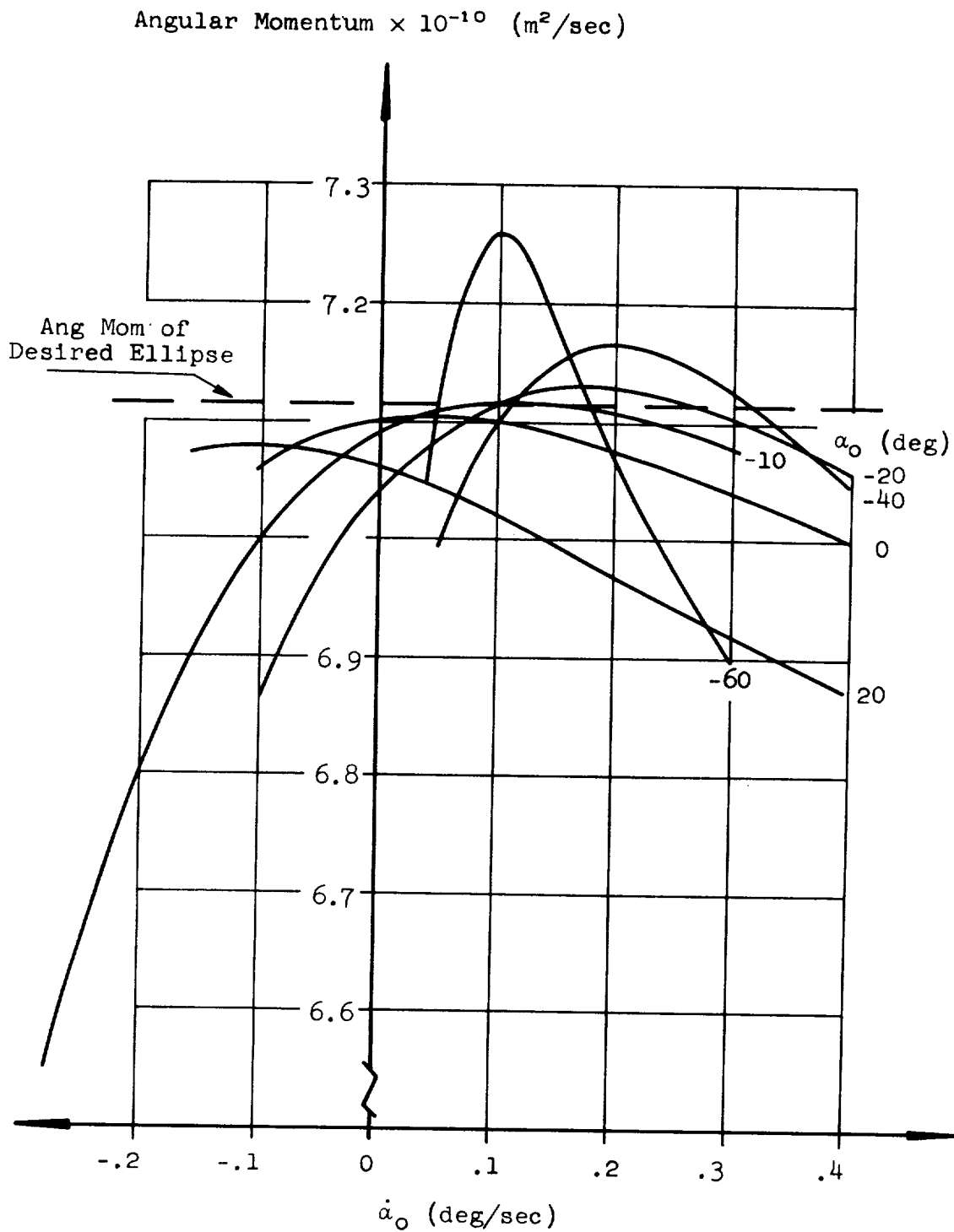
FIG. 4.   VARIATION OF TERMINAL ANGULAR MOMENTUM

WITH $\dot{\alpha}_o$ FOR PARAMETERS OF α_o

FOR F/W$_o$ = 1; y$_o$ = 100 km; I$_{sp}$ = 700 sec
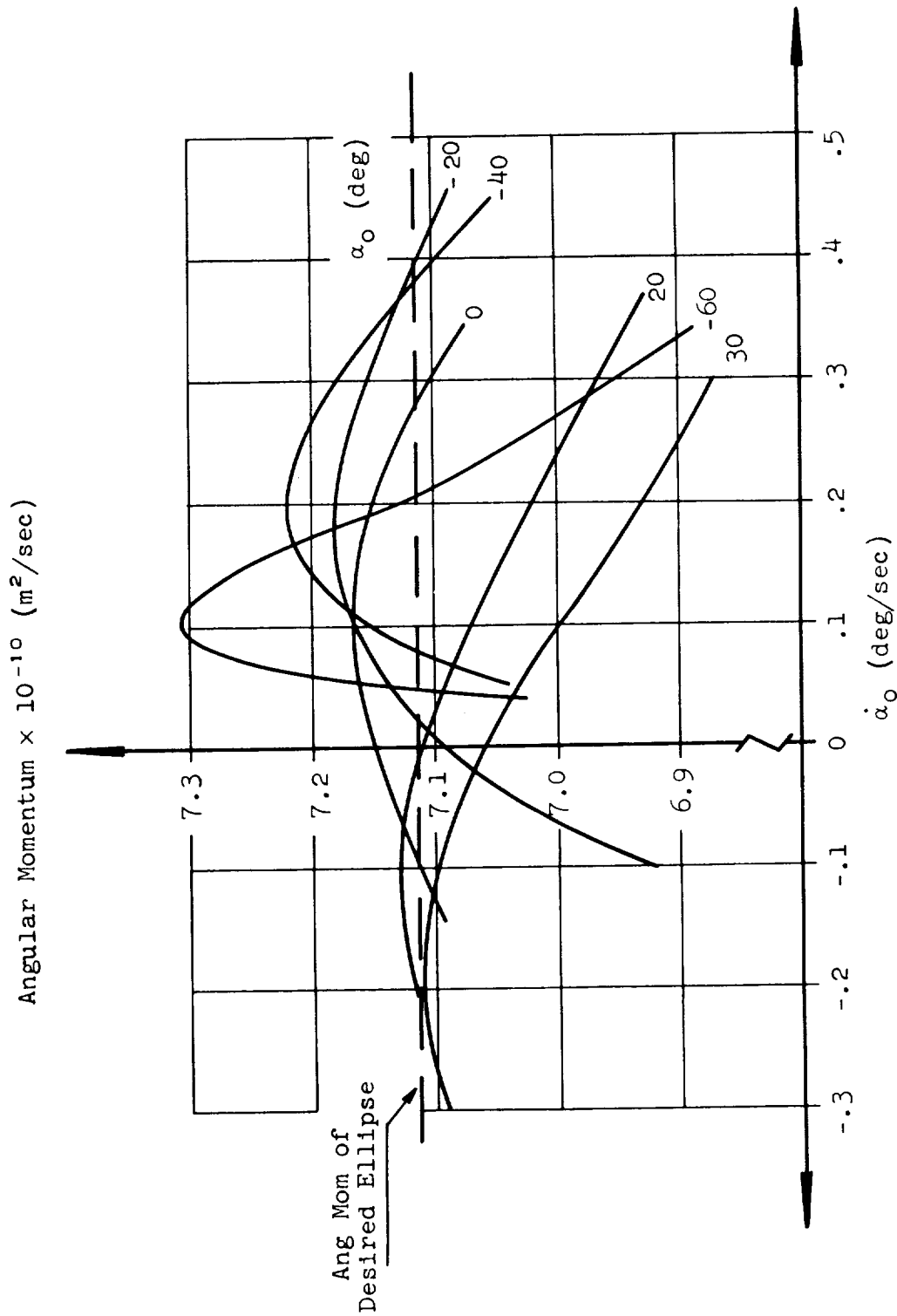
FIG. 5. VARIATION OF TERMINAL ANGULAR MOMENTUM WITH $\dot{\alpha}_o$ FOR PARAMETERS OF α_o

FOR $F/W_o = 1$; $y_o = 200$ km; $I_{sp} = 700$ sec
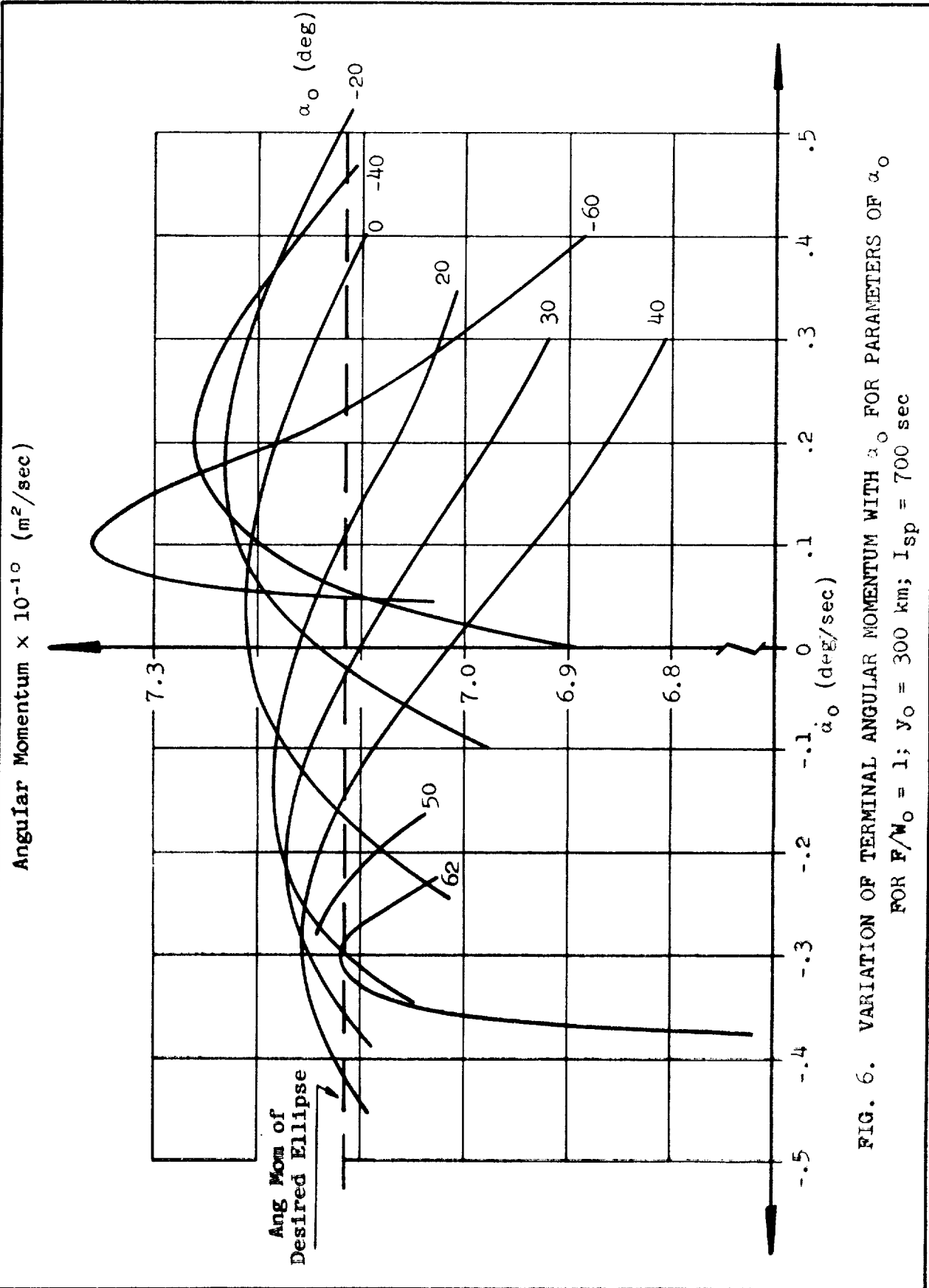
FIG. 6.   VARIATION OF TERMINAL ANGULAR MOMENTUM WITH $\dot{\alpha}_o$ FOR PARAMETERS OF α_o

FOR $F/W_o = 1$; $y_o = 300$ km; $I_{sp} = 700$ sec
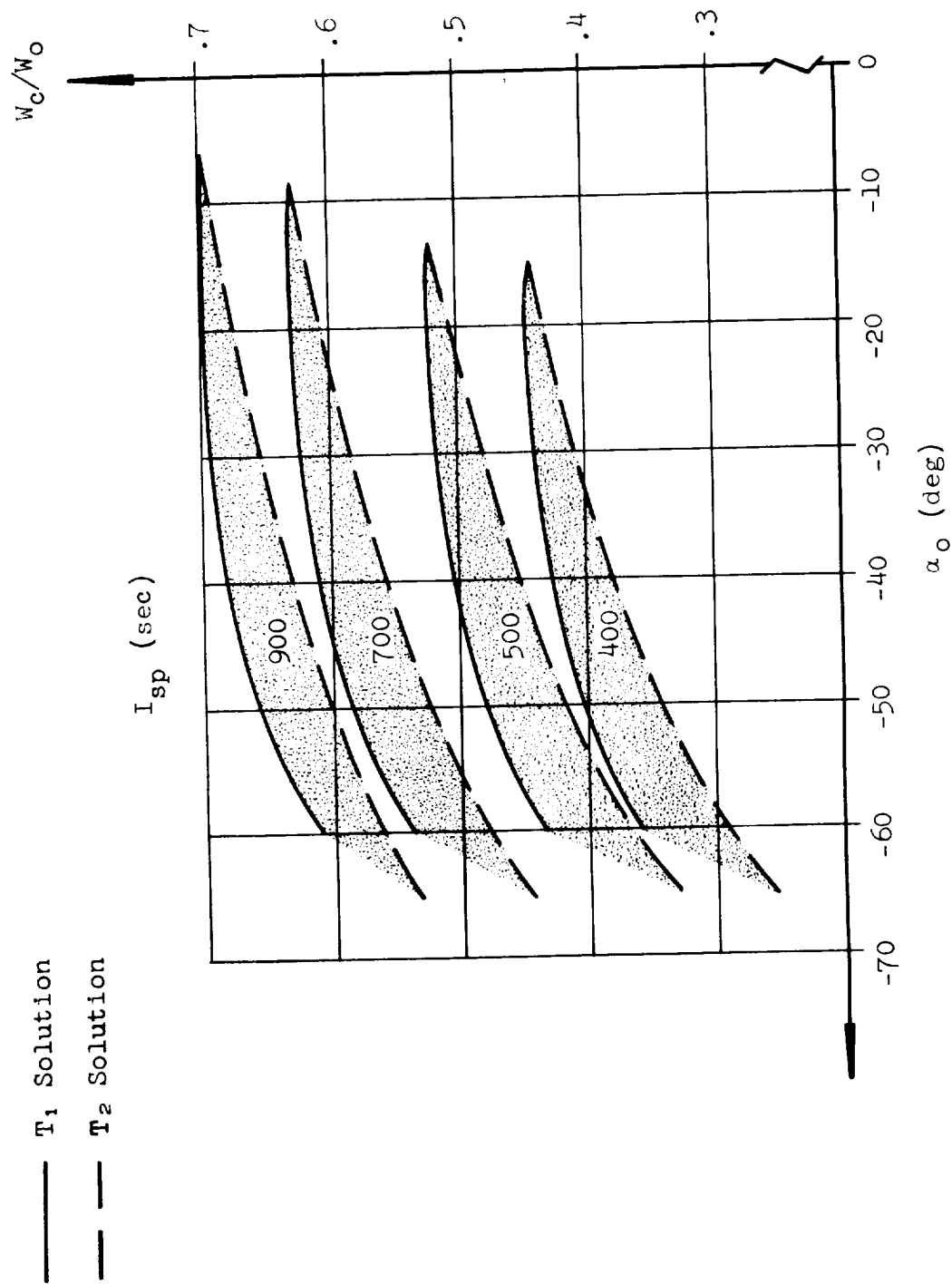
FIG. 7. RATIO OF THE CUTOFF WEIGHT TO THE INITIAL WEIGHT AS A FUNCTION OF α_o

FOR PARAMETERS OF I_{sp} WITH $F/W_o = 1$; $y_o = 100$ km
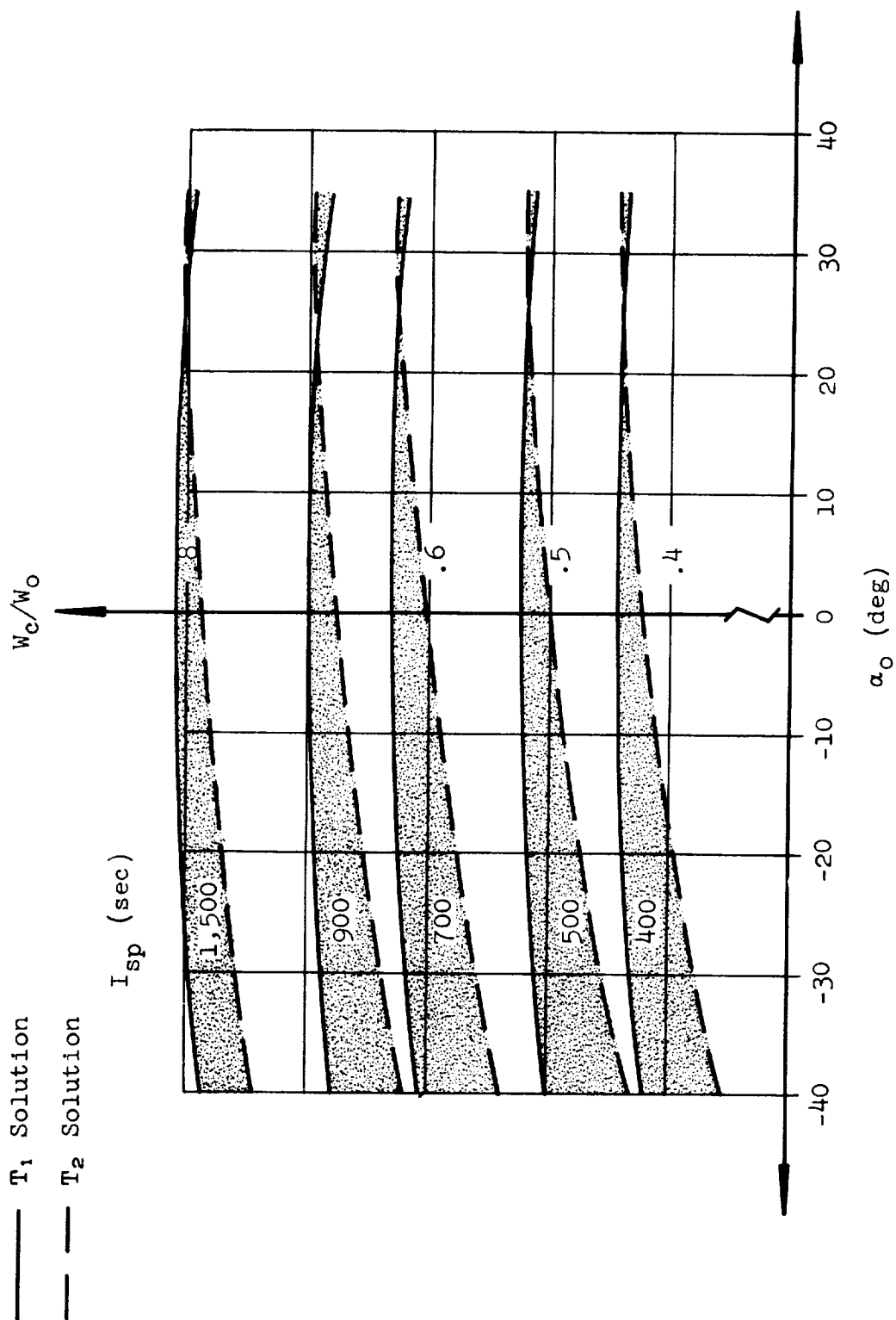
FIG. 8. RATIO OF THE CUTOFF WEIGHT TO THE INITIAL WEIGHT AS A FUNCTION OF α_o

FOR PARAMETERS OF I_{sp} WITH $F/W_o = 1$; $y_o = 200$ km
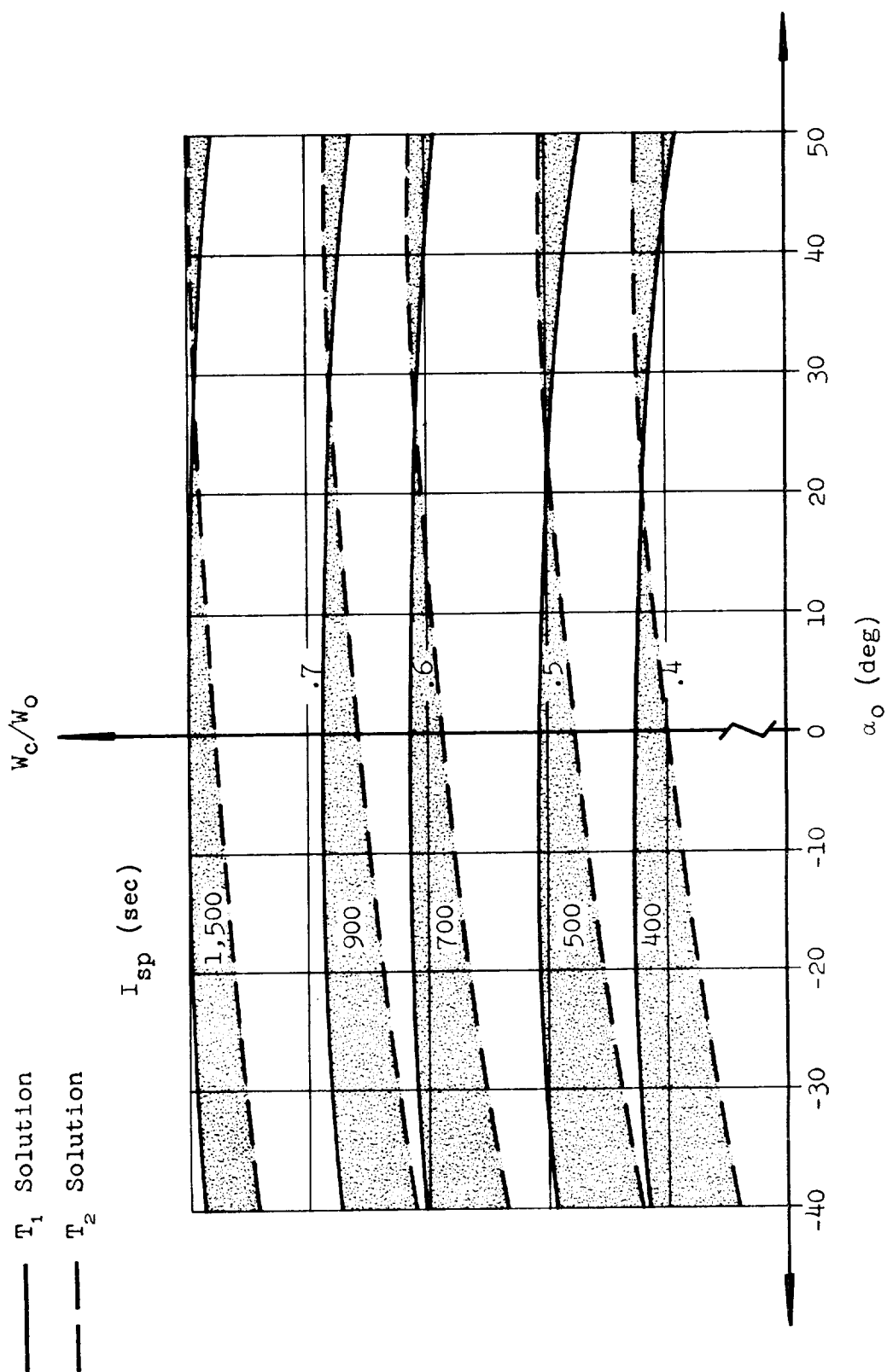
FIG. 9. RATIO OF THE CUTOFF WEIGHT TO THE INITIAL WEIGHT AS A FUNCTION OF α_o

FOR PARAMETERS OF I_{sp} WITH $F/W_o = 1$; $y_o = 300$ km
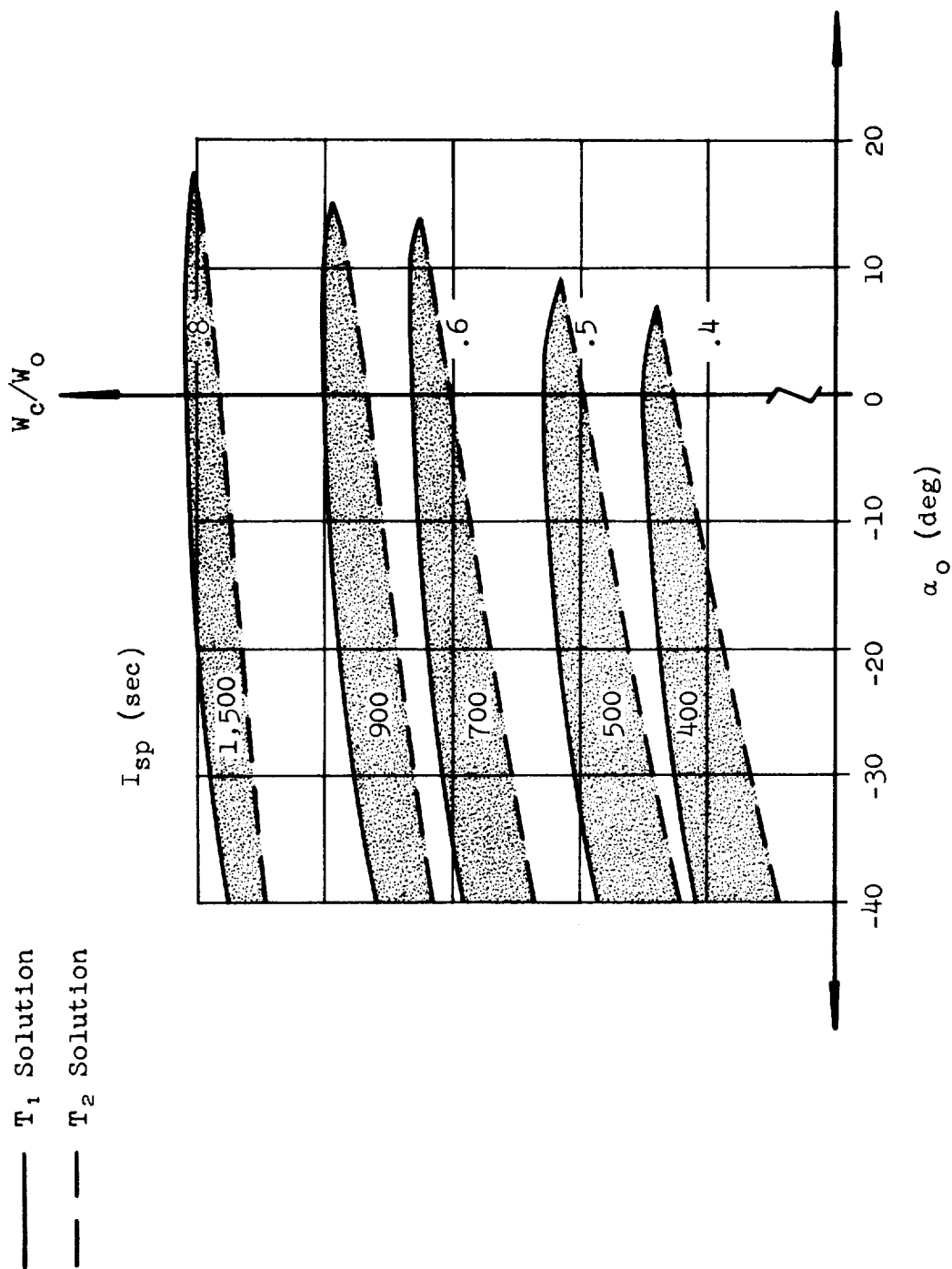
FIG. 10. RATIO OF THE CUTOFF WEIGHT TO THE INITIAL WEIGHT AS A FUNCTION OF α_o

FOR PARAMETERS OF I_{sp} WITH $F/W_o = .5$; $y_o = 100$ km
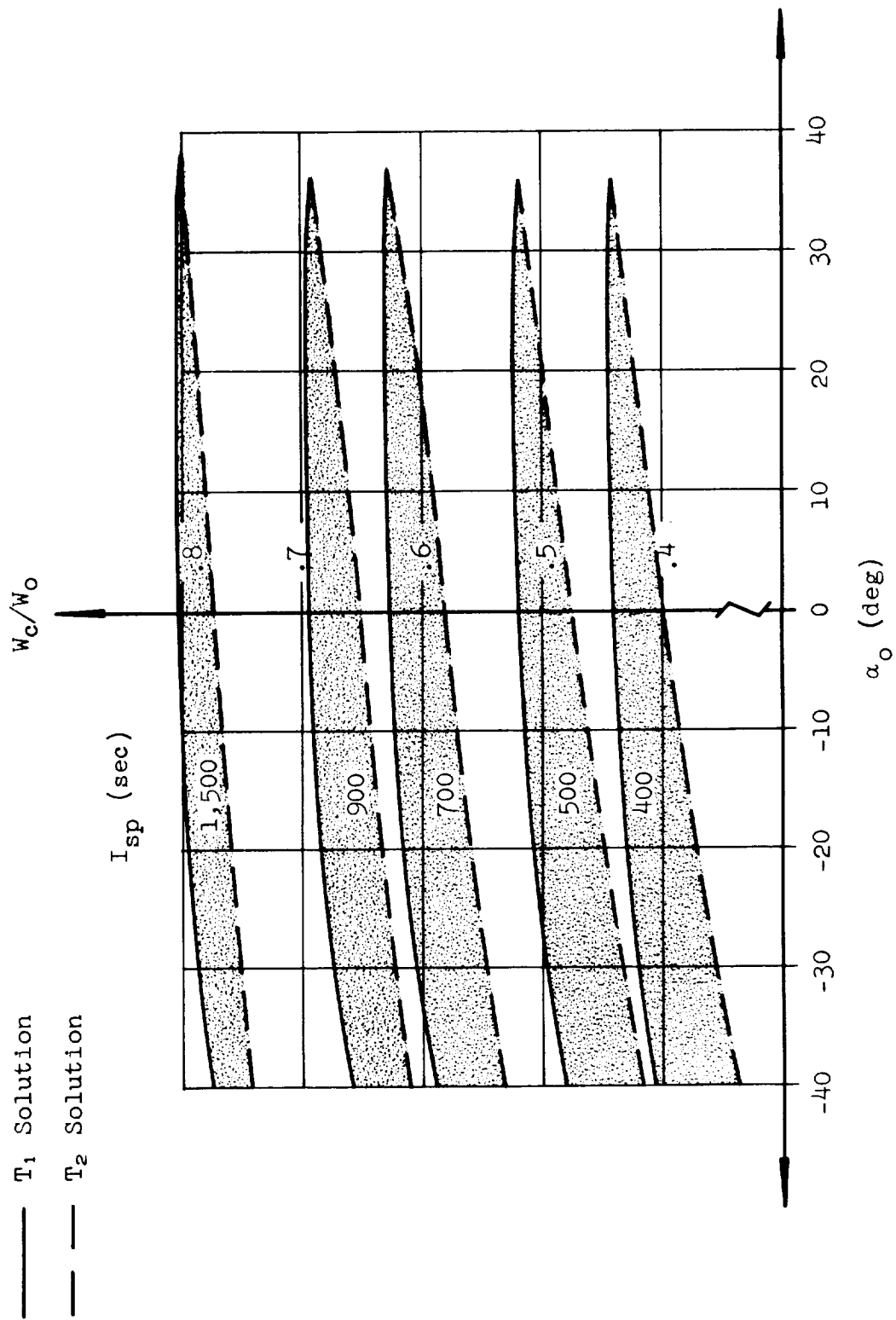
FIG. 11. RATIO OF THE CUTOFF WEIGHT TO THE INITIAL WEIGHT AS A FUNCTION OF α_o

FOR PARAMETERS OF I_{sp} WITH $F/W_o = .5$; $y_o = 200$ km
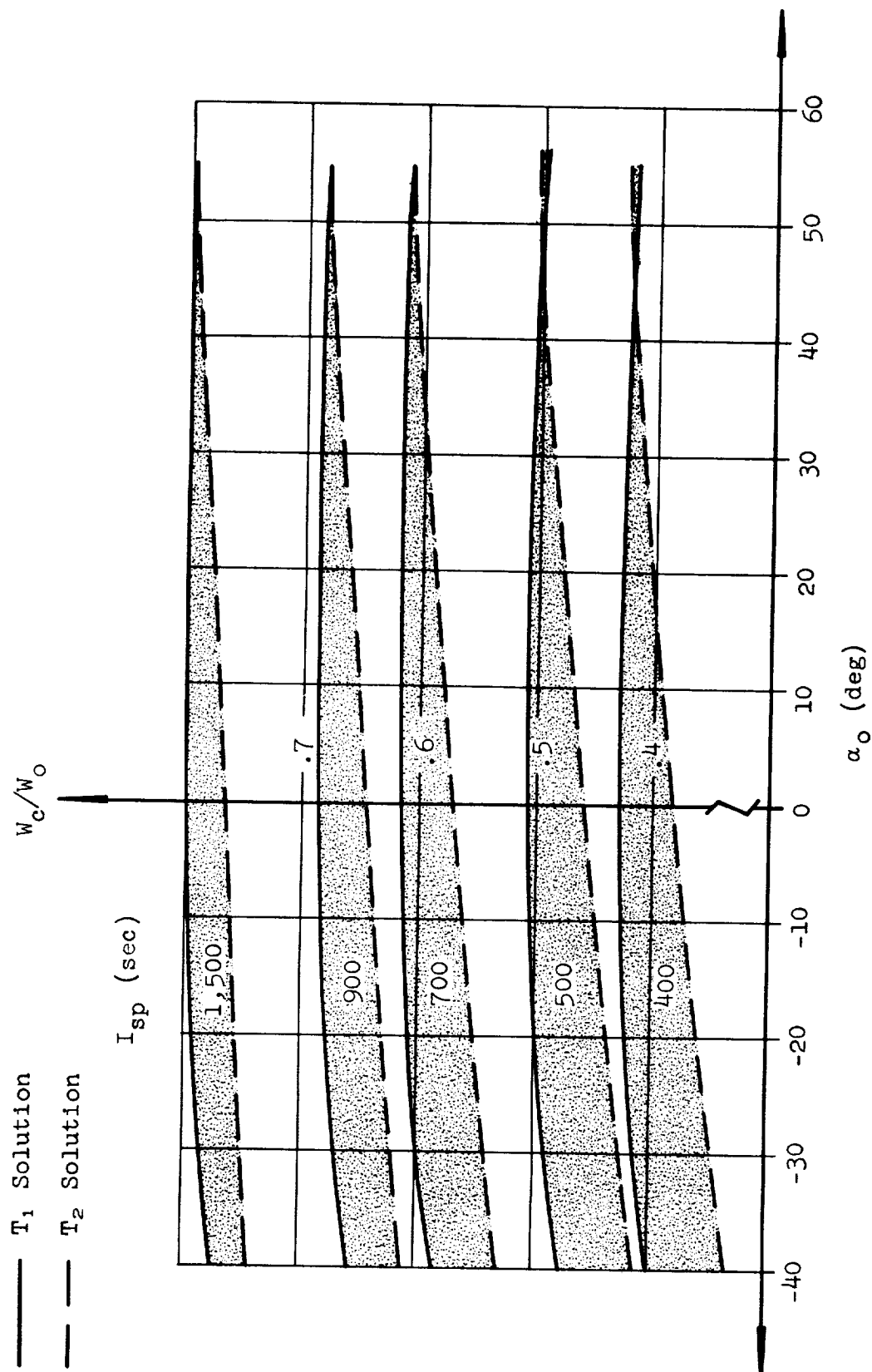
FIG. 12. RATIO OF THE CUTOFF WEIGHT TO THE INITIAL WEIGHT AS A FUNCTION OF α_o FOR PARAMETERS OF I_{sp} WITH $F/W_o = .5$; $y_o = 300$ km
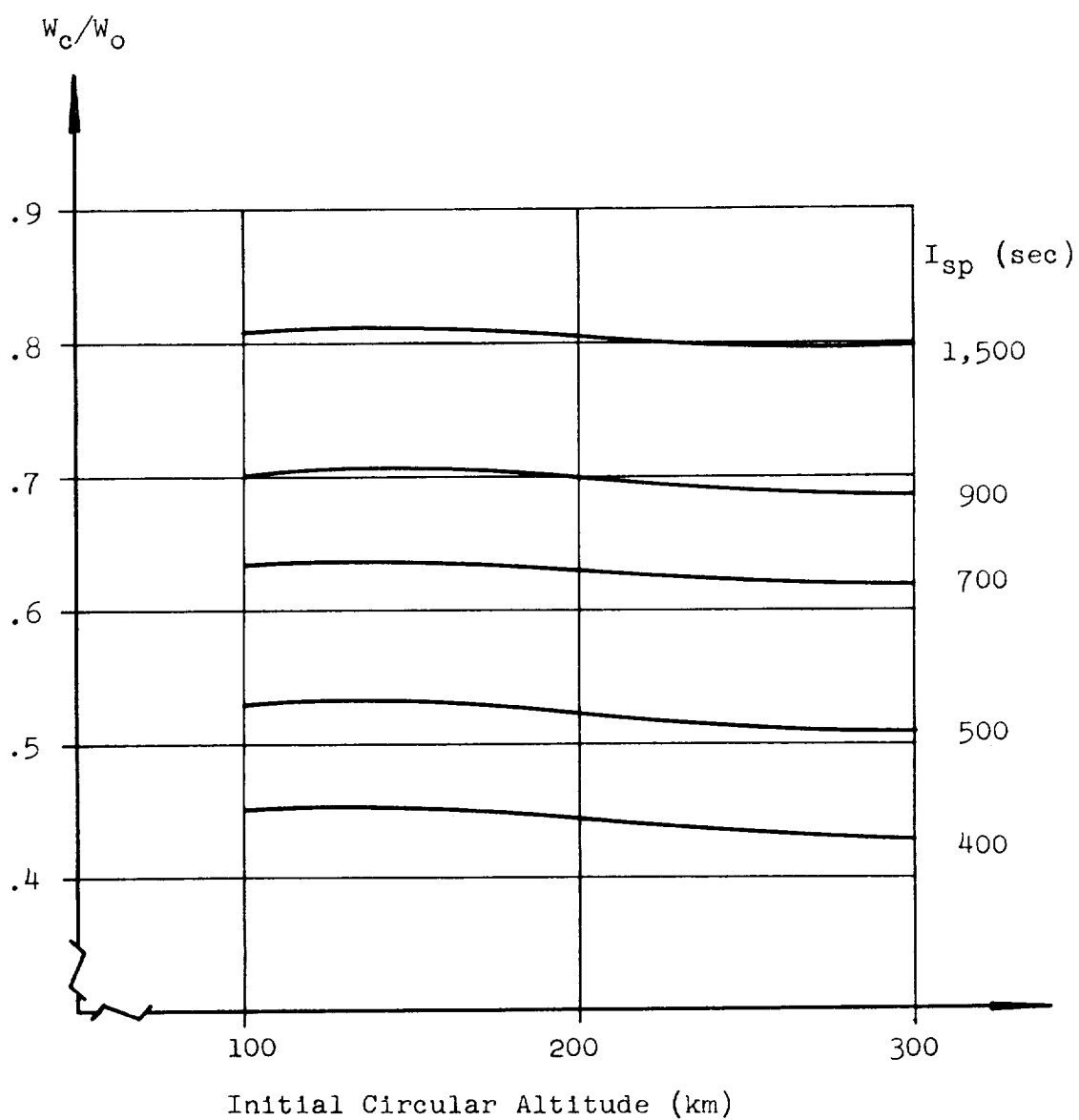
FIG. 13. RATIO OF THE CUTOFF WEIGHT TO THE INITIAL WEIGHT
AS A FUNCTION OF y_o FOR THE **OPTIMUM** α_o AND
THE T_1 SOLUTIONS FOR F/W_o = .5 AND PARAMETERS OF I_{sp}

W_c/W_o

.9

I_{sp} (sec)

.8 ———————————————————————— 1,500

.7 ———————————————————————— 900

.6 ———————————————————————— 700

.5 ———————————————————————— 500

———————————————————————— 400

.4

100   200   300

Initial Circular Altitude (km)
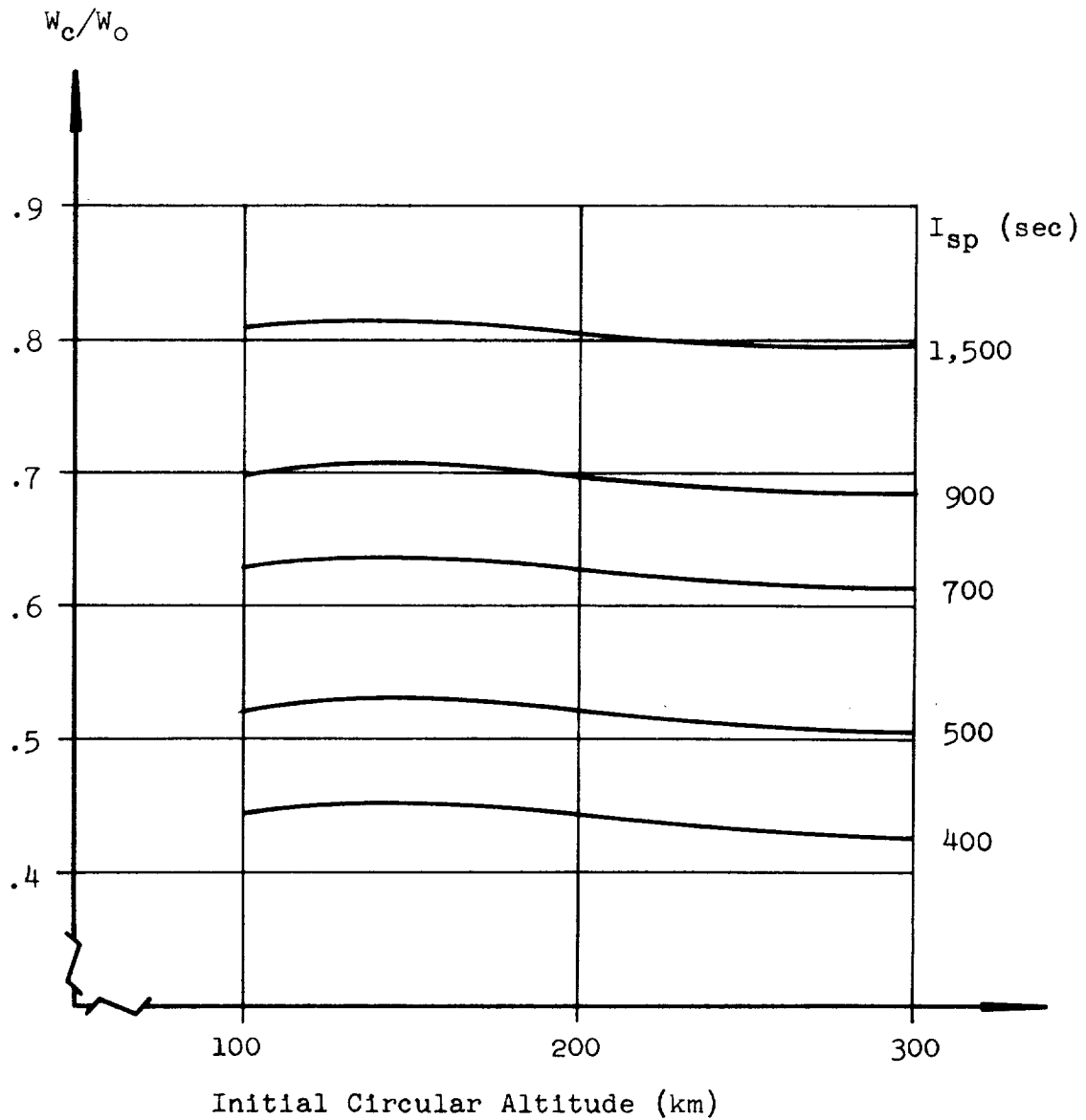
FIG. 14. RATIO OF THE CUTOFF WEIGHT TO THE INITIAL WEIGHT

AS A FUNCTION OF y_o FOR THE OPTIMUM α_o AND

THE T_1 SOLUTIONS FOR $F/W_o = 1$ AND PARAMETERS OF I_{sp}
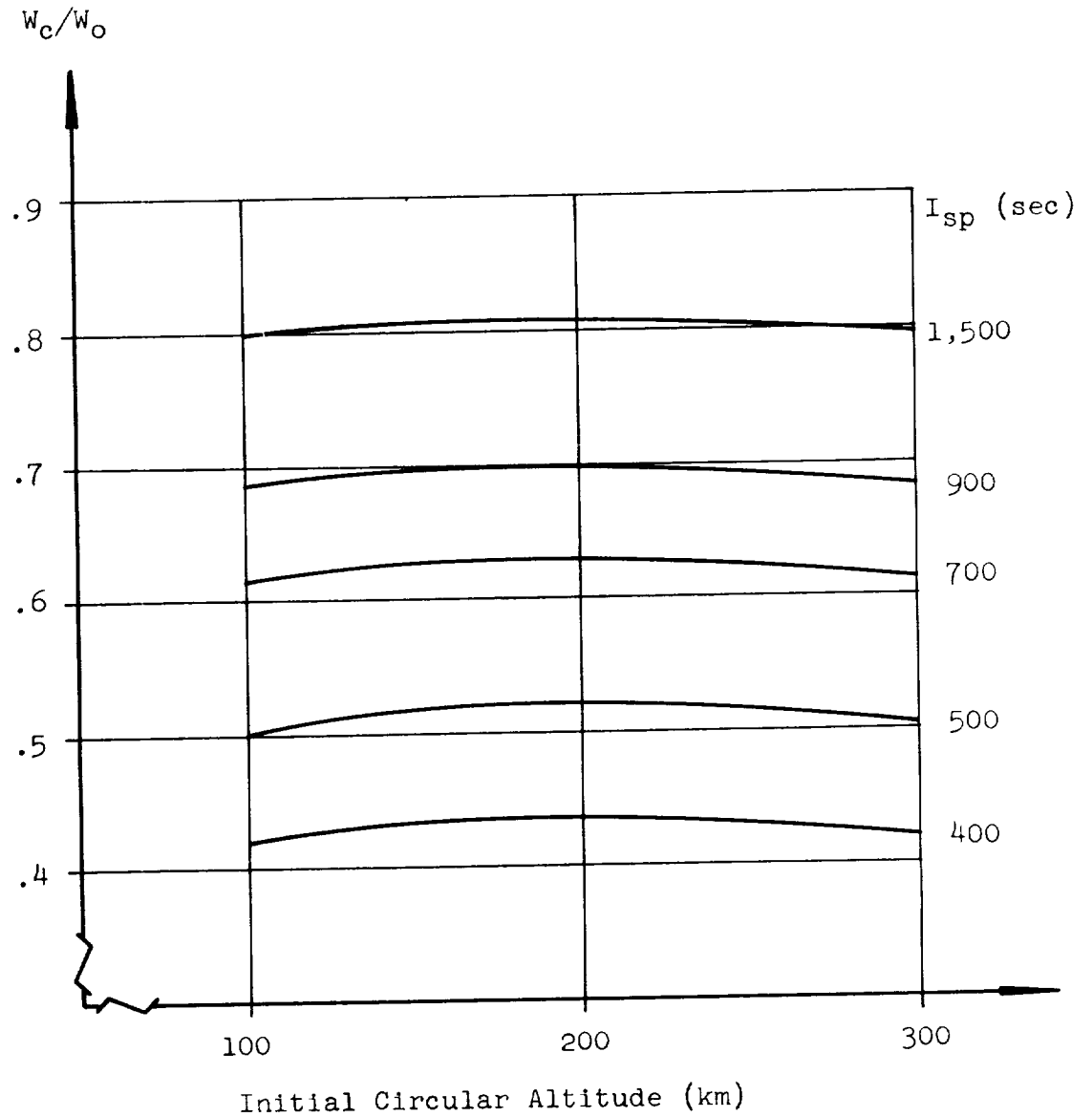
W_c/W_o

FIG. 15. RATIO OF THE CUTOFF WEIGHT TO THE INITIAL WEIGHT
AS A FUNCTION OF y_o FOR THE OPTIMUM α_o AND
THE T_1 SOLUTIONS FOR F/W_o = 2 AND PARAMETERS OF I_{sp}
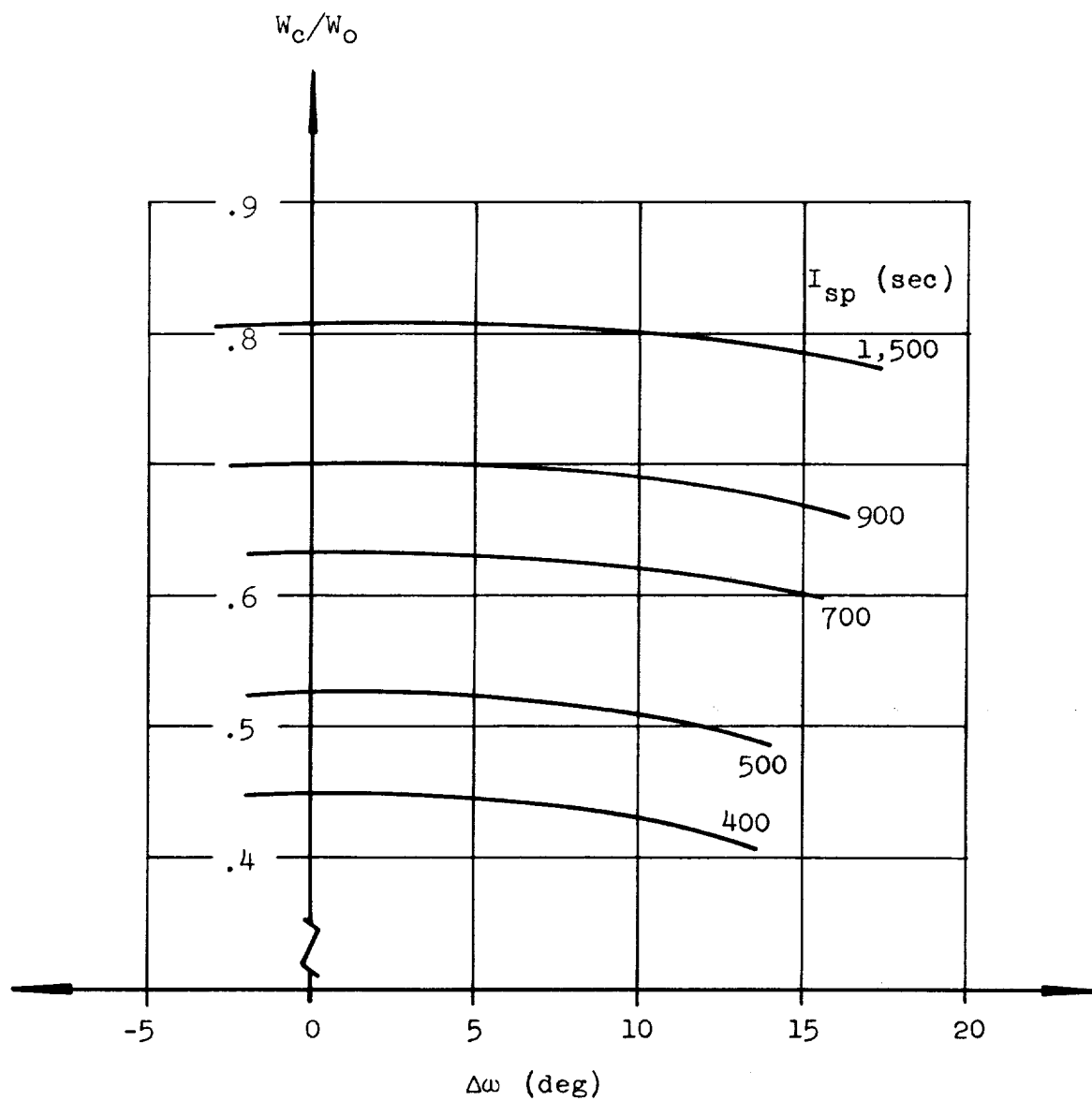
FIG. 16. W_c/W_o AS A FUNCTION OF THE DEVIATION OF
THE DEPARTURE ANGLE ω FROM THE OPTIMUM ω FOR THE T_1 SOLUTIONS
WITH $F/W_o = .5$; $y_o = 100$ km
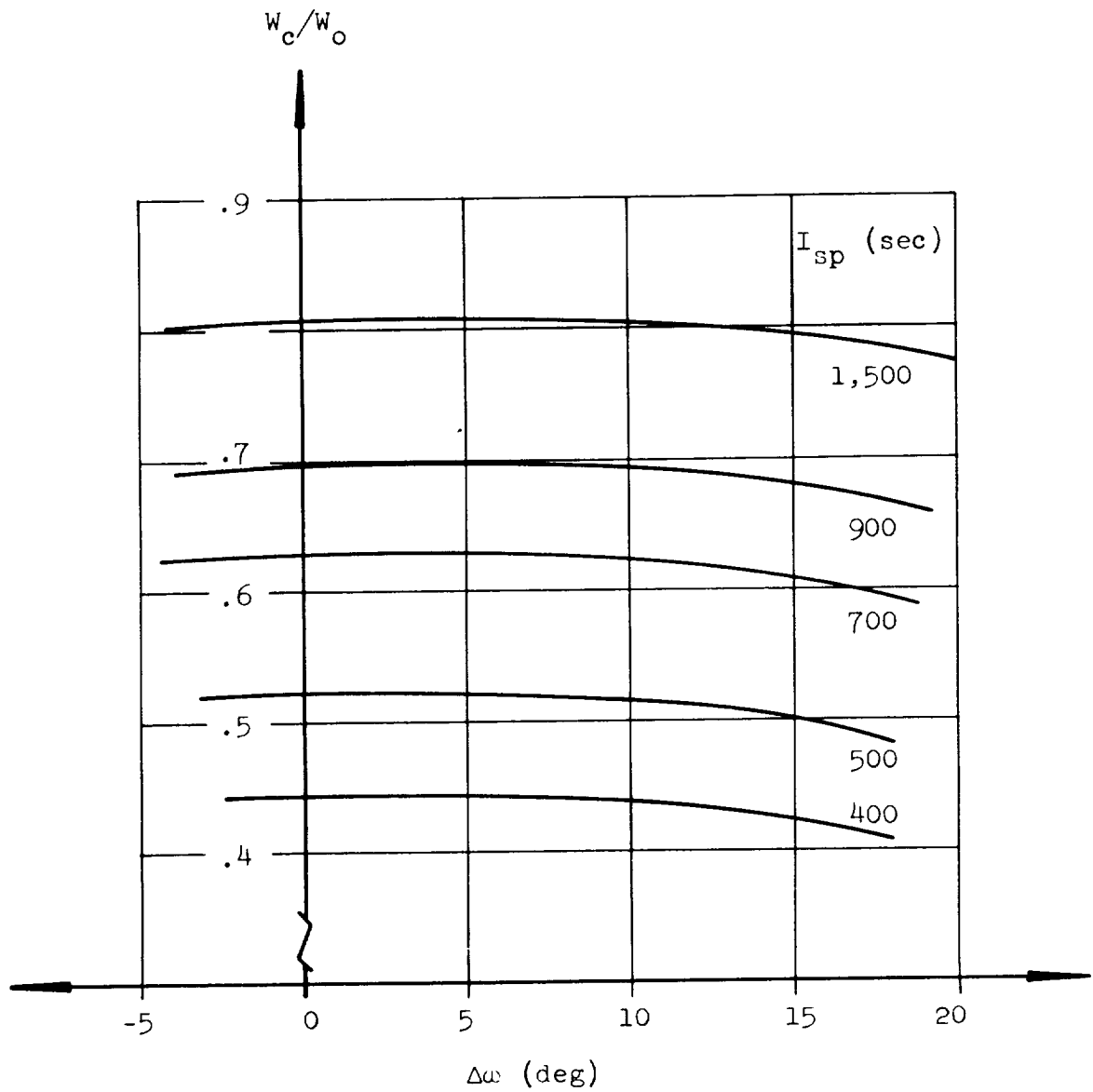
FIG. 17. W_c/W_o AS A FUNCTION OF THE DEVIATION OF
THE DEPARTURE ANGLE ω FROM THE OPTIMUM ω FOR THE T_1 SOLUTIONS
WITH $F/W_o = .5$; $y_o = 200$ km
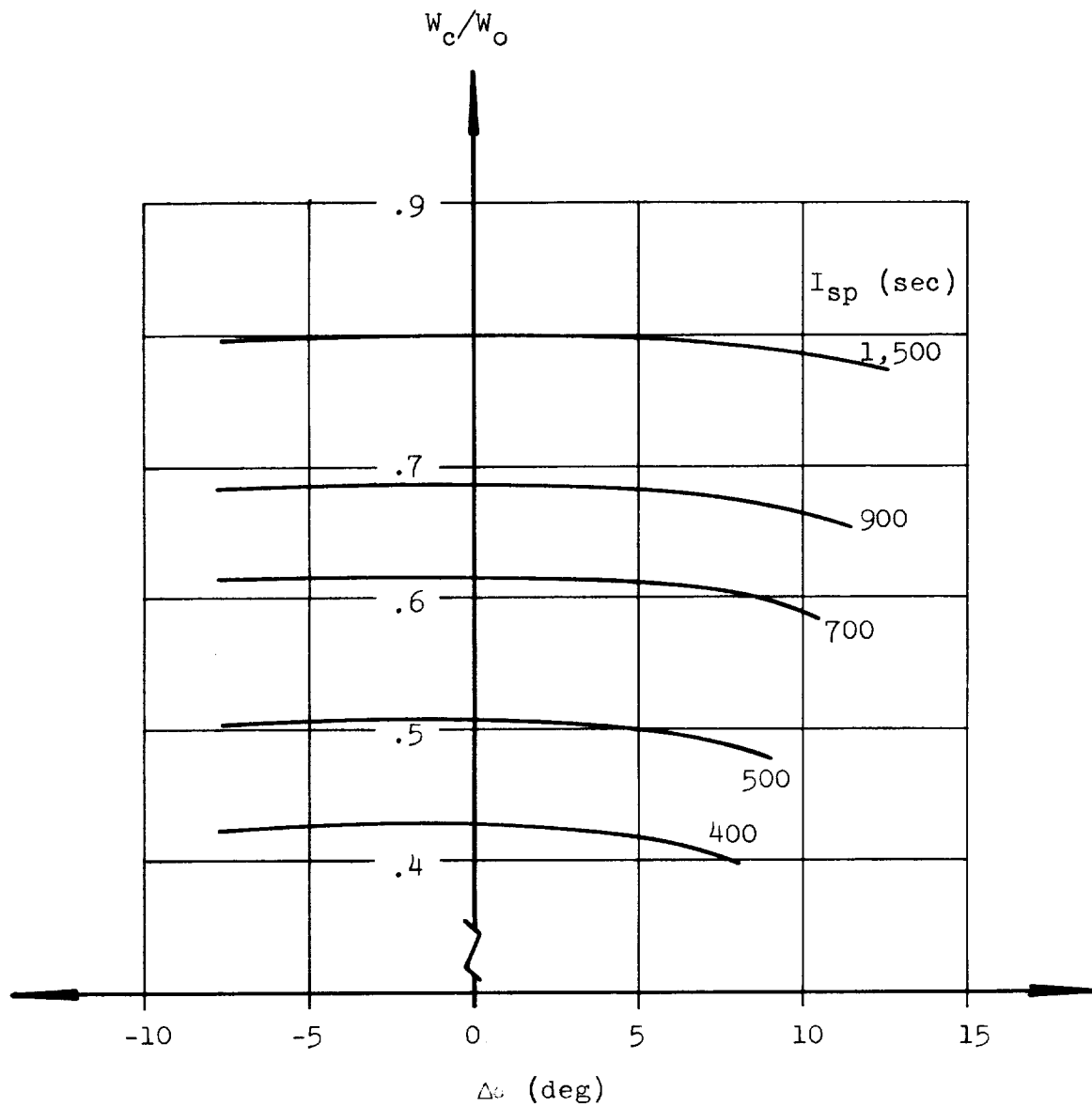
FIG. 18. W_c/W_o AS A FUNCTION OF THE DEVIATION OF
THE DEPARTURE ANGLE FROM THE OPTIMUM FOR THE T_1 SOLUTIONS
WITH $F/W_o = .5$; $y_o = 300$ km
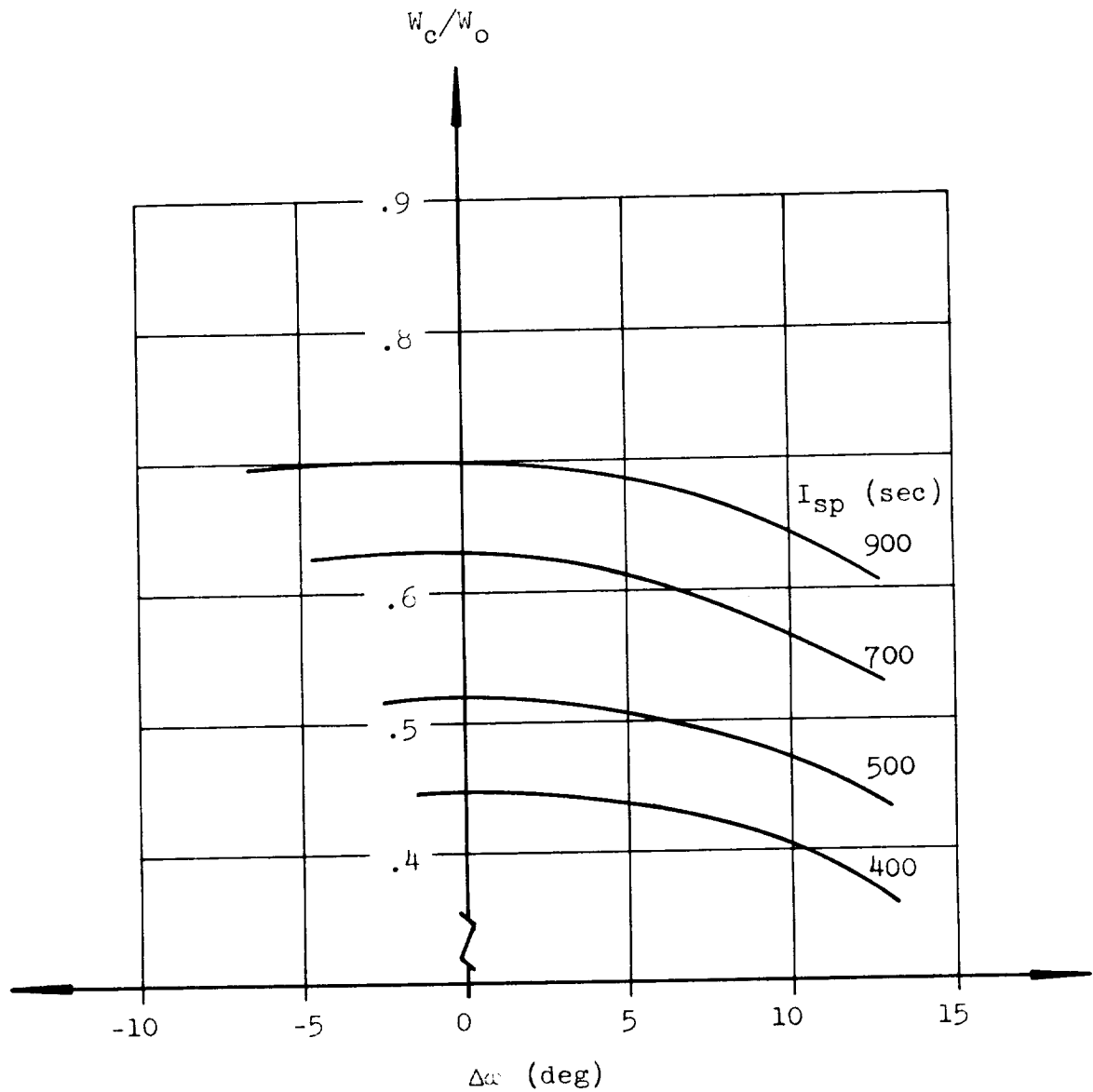
FIG. 19. W_c/W_o AS A FUNCTION OF THE DEVIATION OF
THE DEPARTURE ANGLE ω FROM THE OPTIMUM ω FOR THE T_1 SOLUTIONS
WITH $F/W_o = 1$; $y_o = 100$ km

FIG. 20. W_c/W_o AS A FUNCTION OF THE DEVIATION OF
THE DEPARTURE ANGLE ω FROM THE OPTIMUM ω FOR THE T_1 SOLUTIONS
WITH F/W_o = 1; y_o = 200 km

FIG. 21. W_c/W_o AS A FUNCTION OF THE DEVIATION OF
THE DEPARTURE ANGLE ω FROM THE OPTIMUM ω FOR THE T_1 SOLUTIONS
WITH $F/W_o = 1$; $y_o = 300$ km
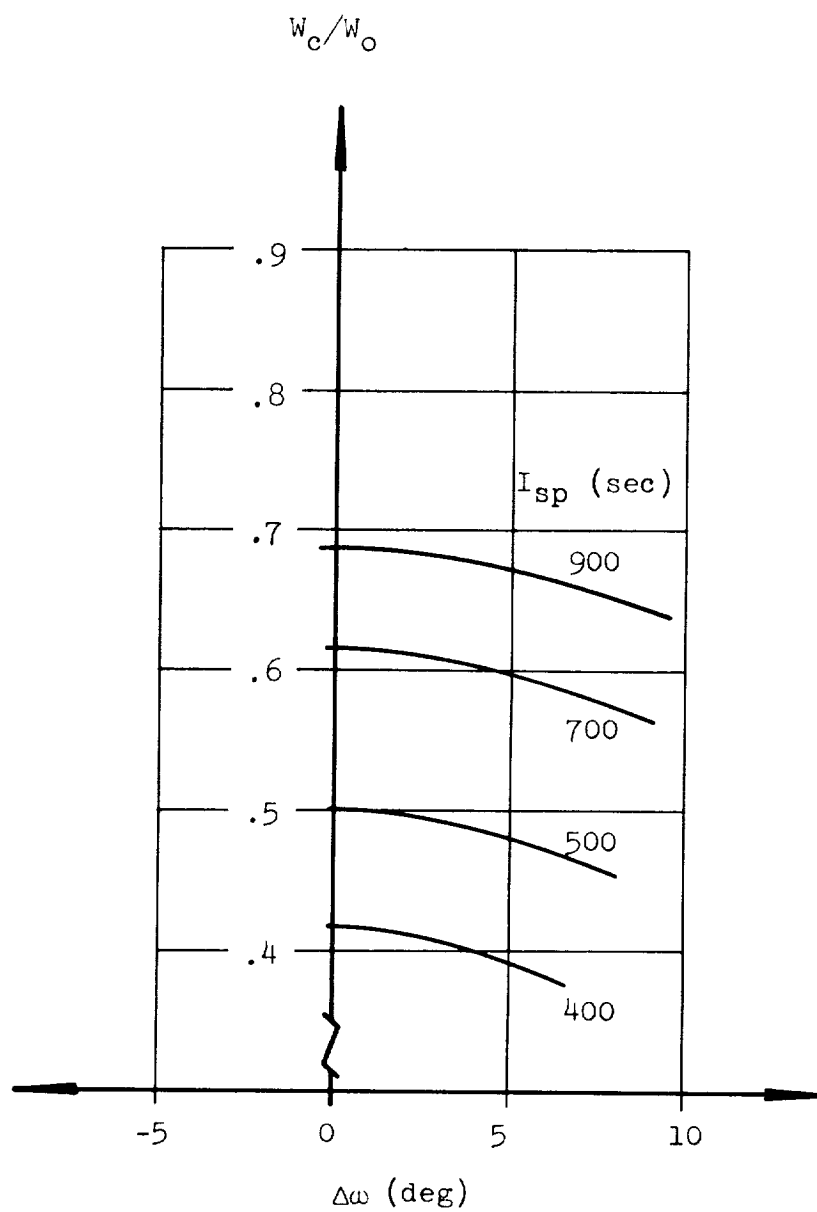
FIG. 22. W_c/W_o AS A FUNCTION OF THE DEVIATION OF
THE DEPARTURE ANGLE ω FROM THE OPTIMUM ω FOR THE T_1 SOLUTIONS
WITH $F/W_o = 2$; $y_o = 100$ km

FIG. 23. W_c/W_o AS A FUNCTION OF THE DEVIATION OF THE DEPARTURE ANGLE ω FROM THE OPTIMUM ω FOR THE T_1 SOLUTIONS WITH $F/W_o = 2$; $y_o = 200$ km

FIG. 24. W_c/W_o AS A FUNCTION OF THE DEVIATION OF
THE DEPARTURE ANGLE ω FROM THE OPTIMUM ω FOR THE T_1 SOLUTIONS
WITH $F/W_o = 2$; $y_o = 300$ km

FIG. 25. W_c/W_o AS A FUNCTION OF THE DEVIATION OF
THE DEPARTURE ANGLE ω FROM THE OPTIMUM ω FOR THE T_2 SOLUTIONS
WITH $F/W_o = 1$; $y_o = 200$ km

FIG. 26. W_c/W_o AS A FUNCTION OF THE DEVIATION OF
THE DEPARTURE ANGLE ω FROM THE OPTIMUM ω FOR THE T_2 SOLUTIONS
WITH $F/W_o = 1$; $y_o = 300$ km

FIG. 27. W_c/W_o AS A FUNCTION OF THE DEVIATION OF
THE DEPARTURE ANGLE ω FROM THE OPTIMUM ω FOR THE T_2 SOLUTIONS
WITH $F/W_o = 2$; $y_o = 200$ km

FIG. 28. W_c/W_o AS A FUNCTION OF THE DEVIATION OF

THE DEPARTURE ANGLE ω FROM THE OPTIMUM ω FOR THE T_2 SOLUTIONS

WITH $F/W_o = 2$; $y_o = 300$ km

FIG. 29. THE RATE OF CHANGE OF THE DEPARTURE ANGLE ω WITH RESPECT TO TIME

AS A FUNCTION OF THE INITIAL CIRCULAR ALTITUDE
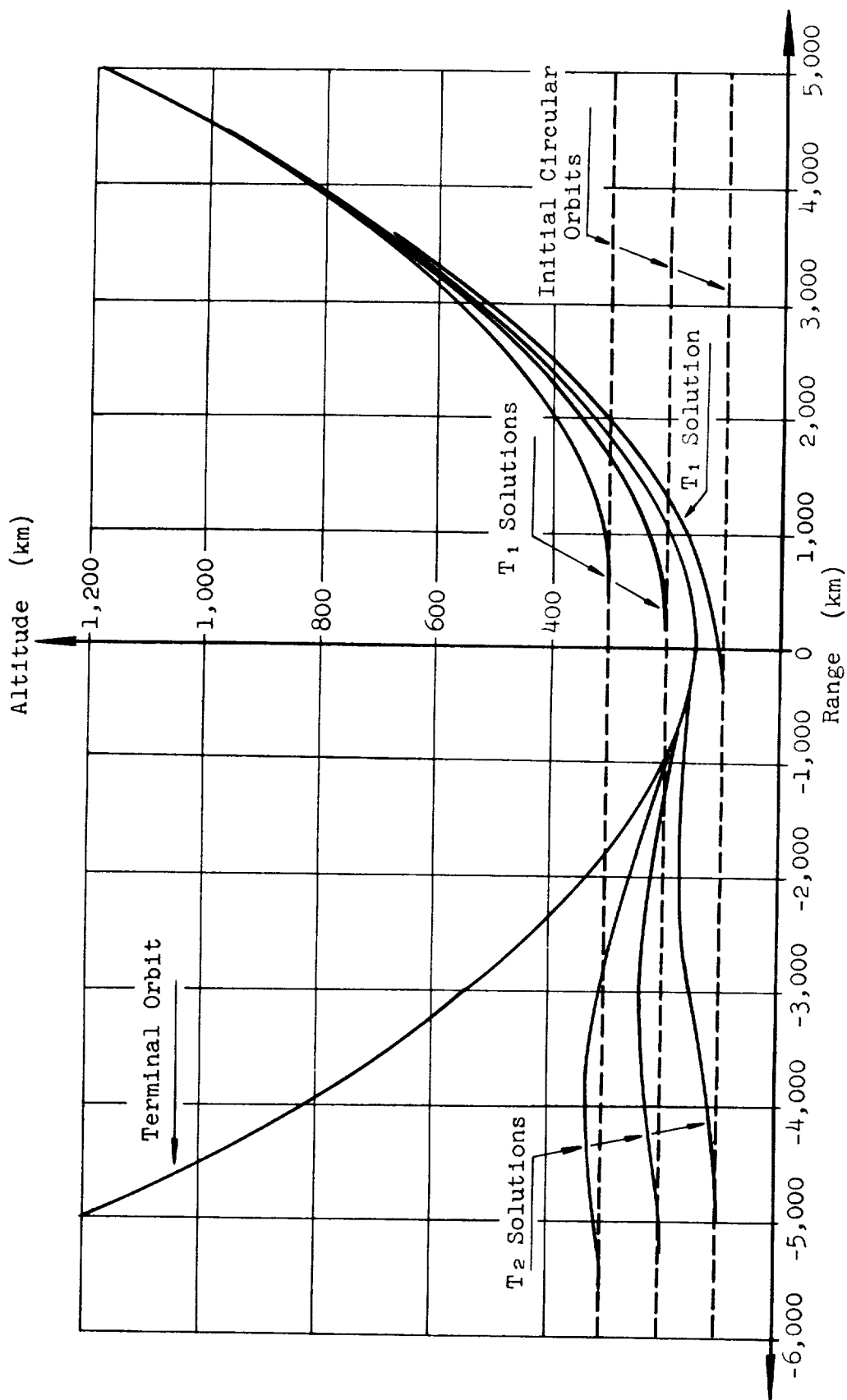
FIG. 30. NON-OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS

FOR $F/W_o = .5$; $I_{sp} = 400$ SEC; $\alpha_o = -40$ DEG

FIG. 31. NON-OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS
FOR $F/W_o = .5$; $I_{sp} = 900$ SEC; $\alpha_o = -40$ DEG
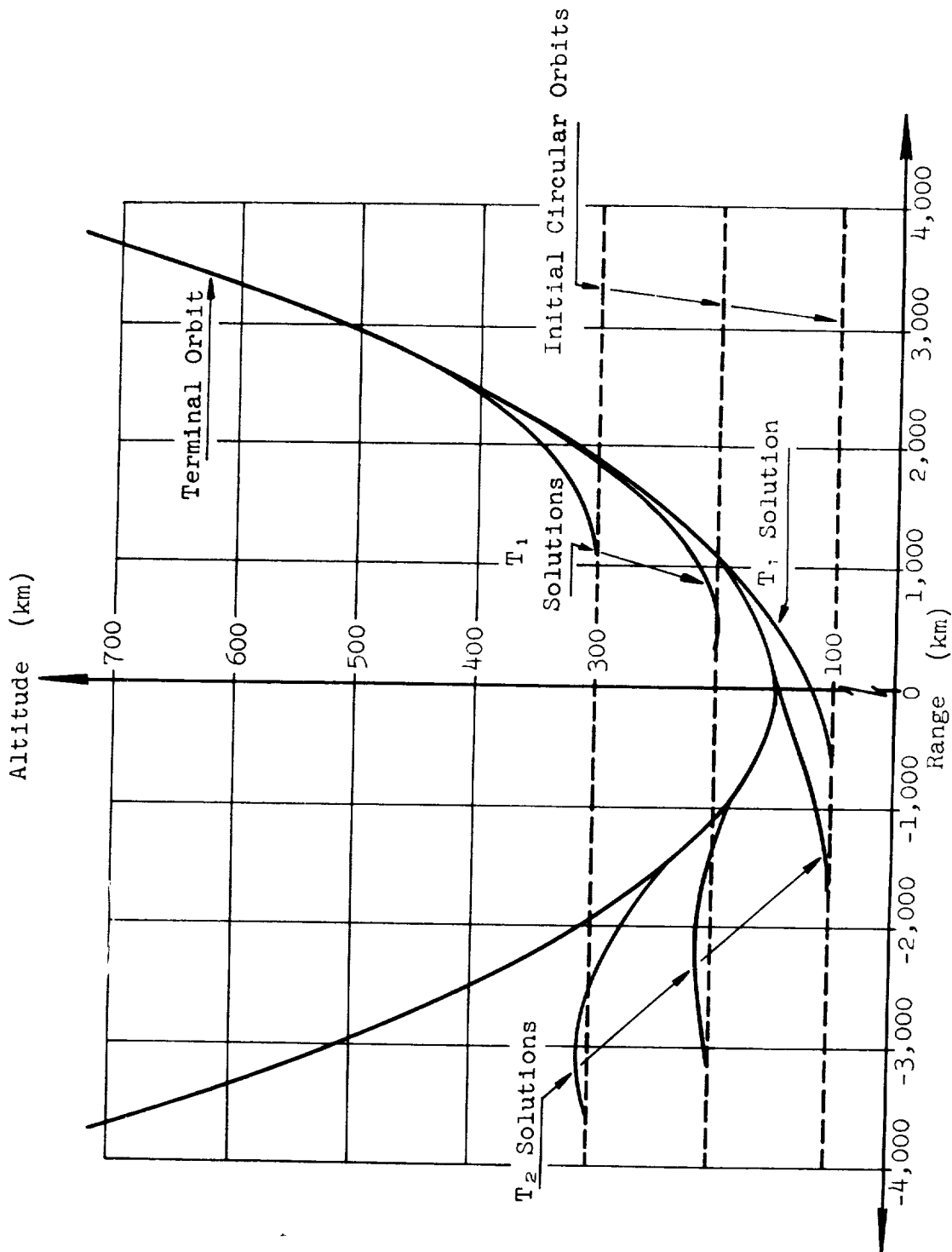
FIG. 32.   NON-OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS
FOR $F/W_o = 1$; $I_{sp} = 400$ SEC; $\alpha_o = -20$ DEG

FIG. 33. NON-OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS FOR $F/W_o = 1$; $I_{sp} = 900$ SEC; $\alpha_o = -20$ DEG

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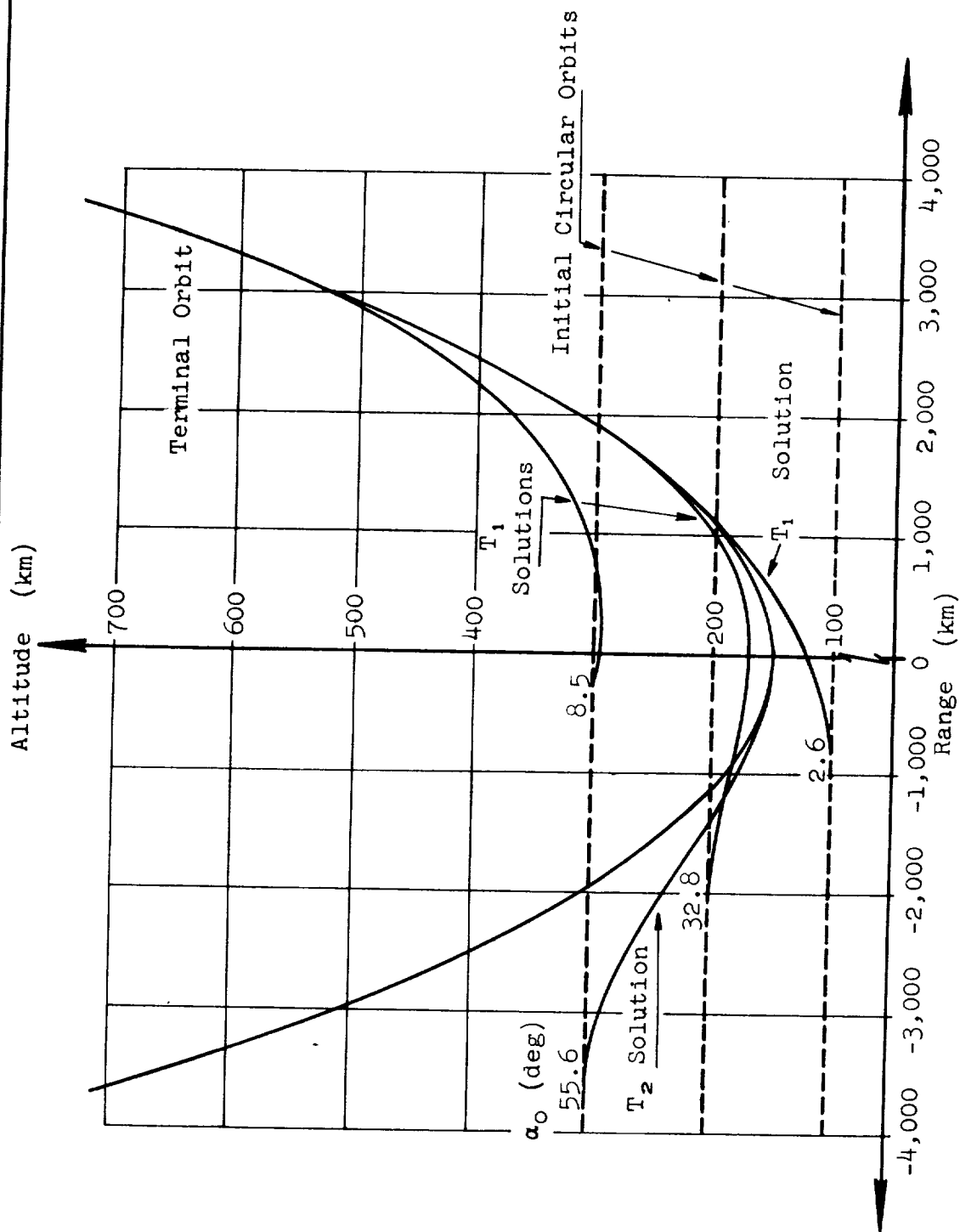
FIG. 34. OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS
FOR $F/W_0 = .5$; $I_{sp} = 400$ SEC
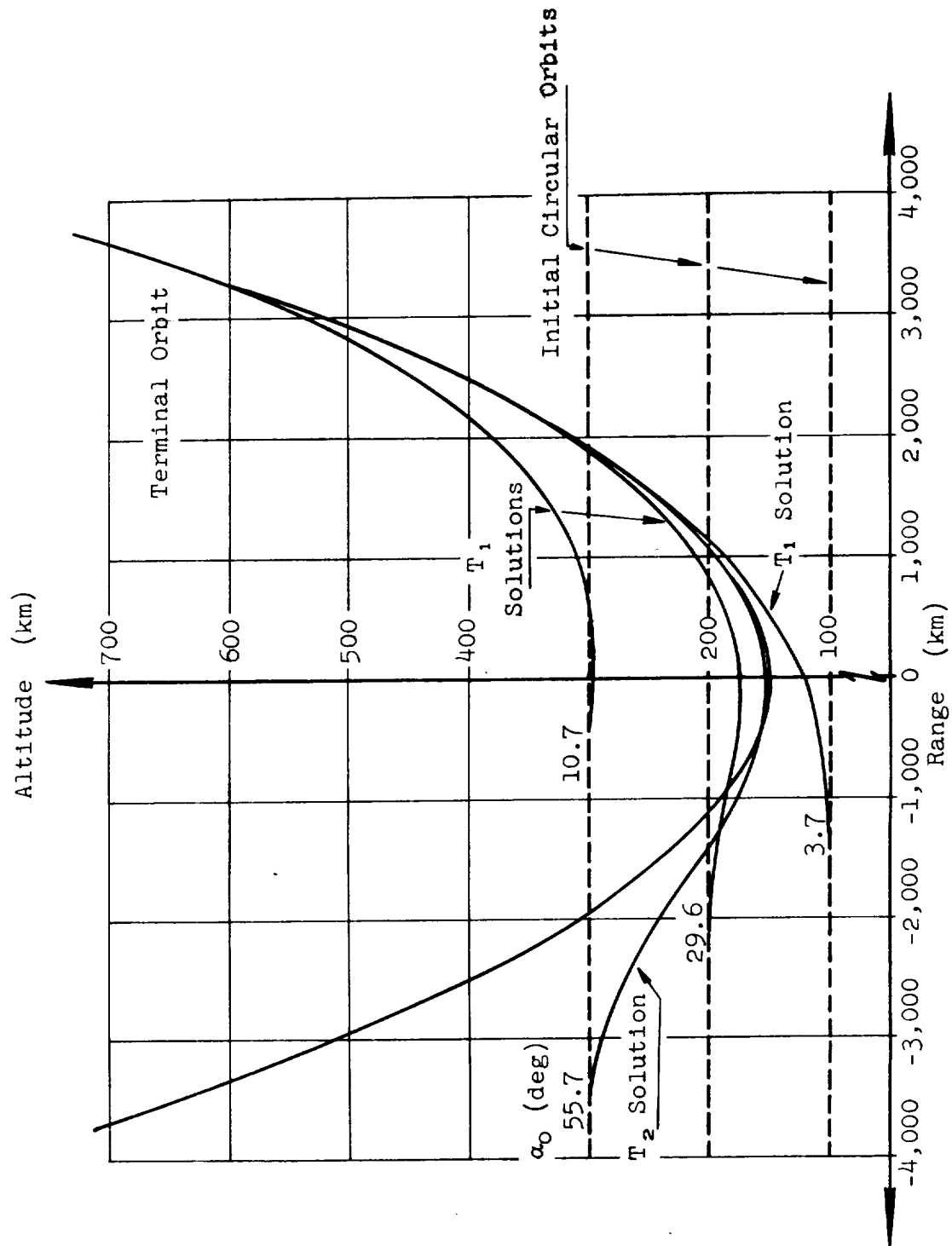
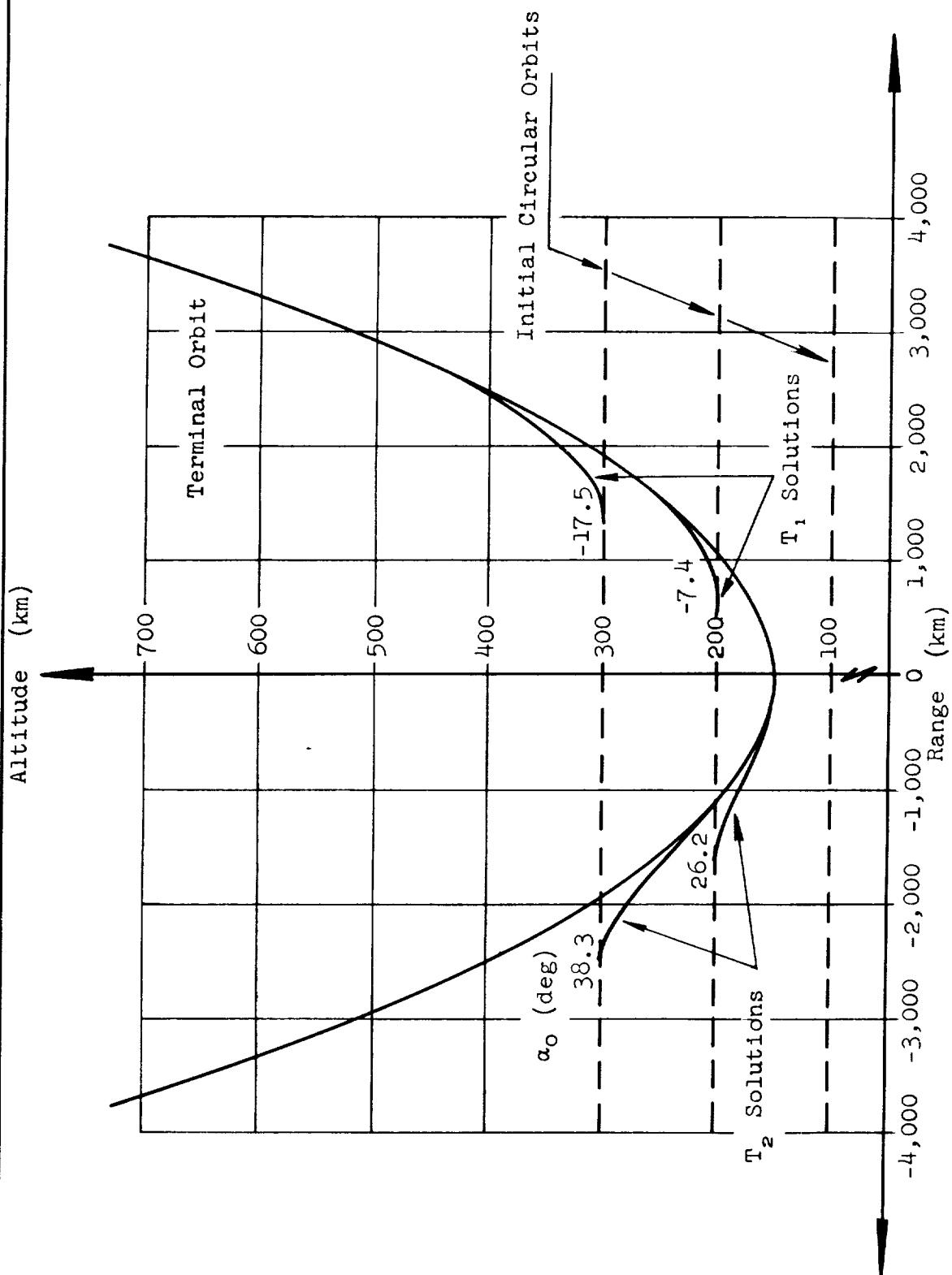FIG. 35. OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS
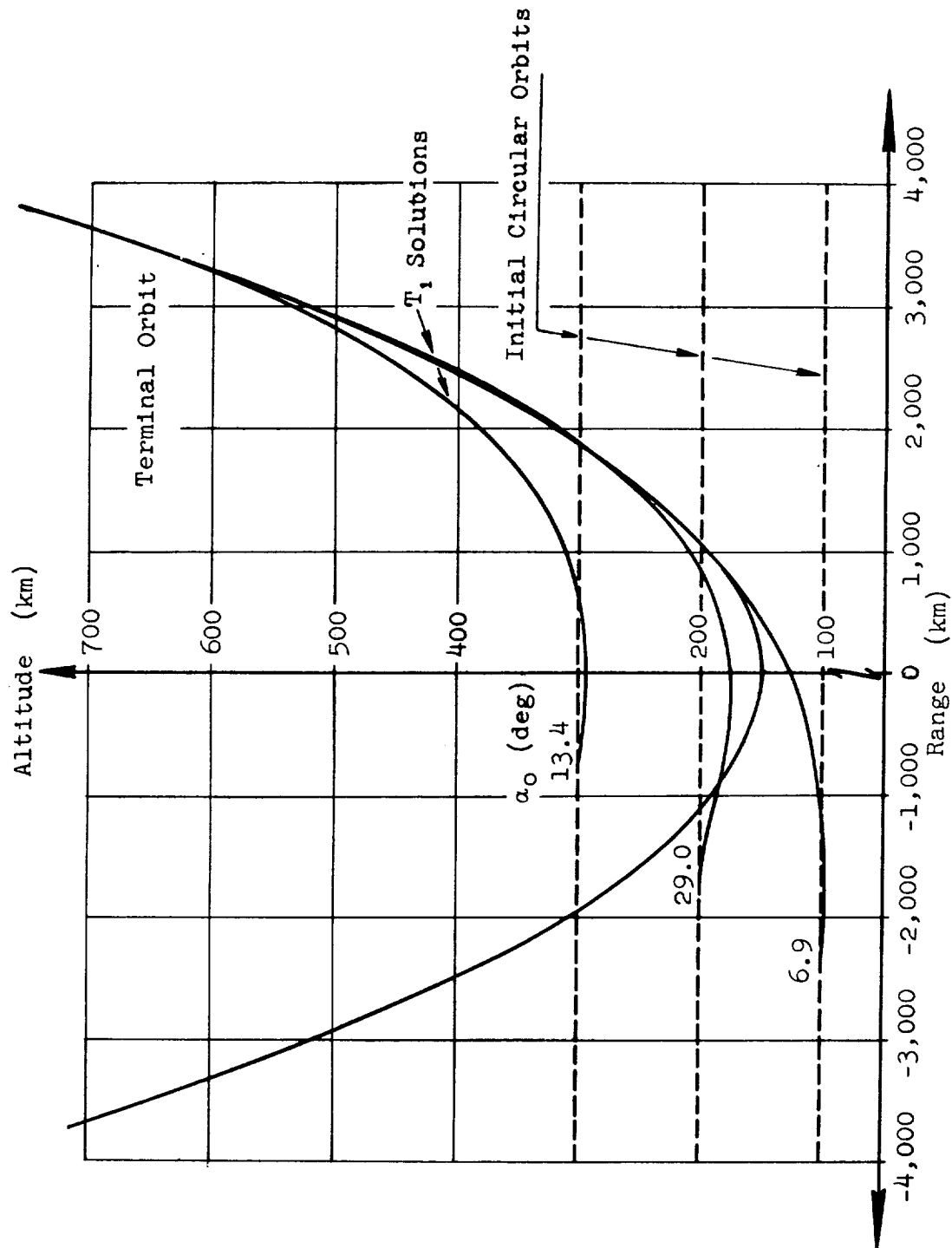
FOR F/Wo = .5; Isp = 500 SEC

FIG. 36. OPTIMUM TRAJECTORIES FOR T_1 SOLUTIONS
FOR $F/W_o = .5$; $I_{sp} = 700$ SEC
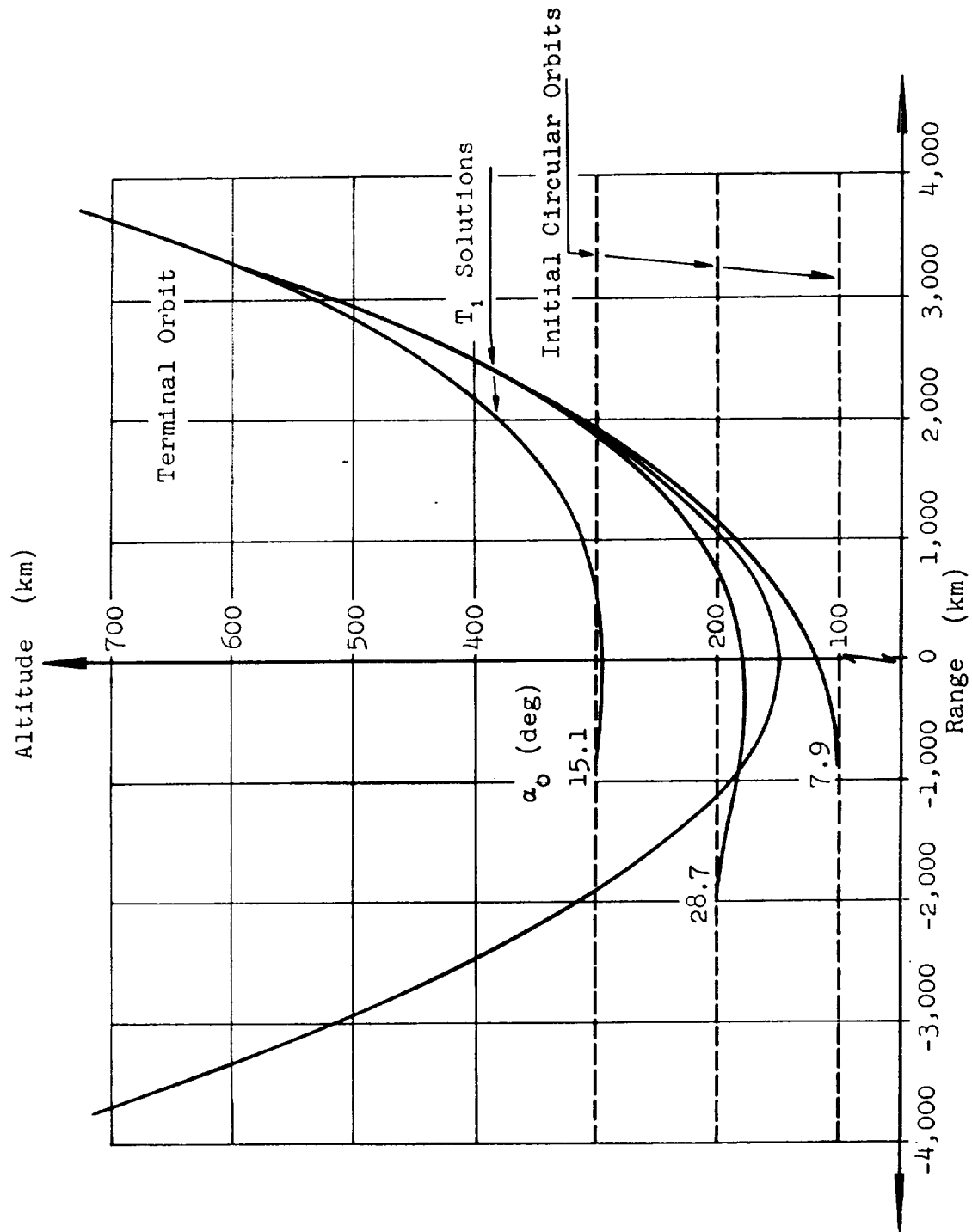
FIG. 37. OPTIMUM TRAJECTORIES FOR T_1 SOLUTIONS
FOR F/W = .5; I_{sp} = 900 SEC
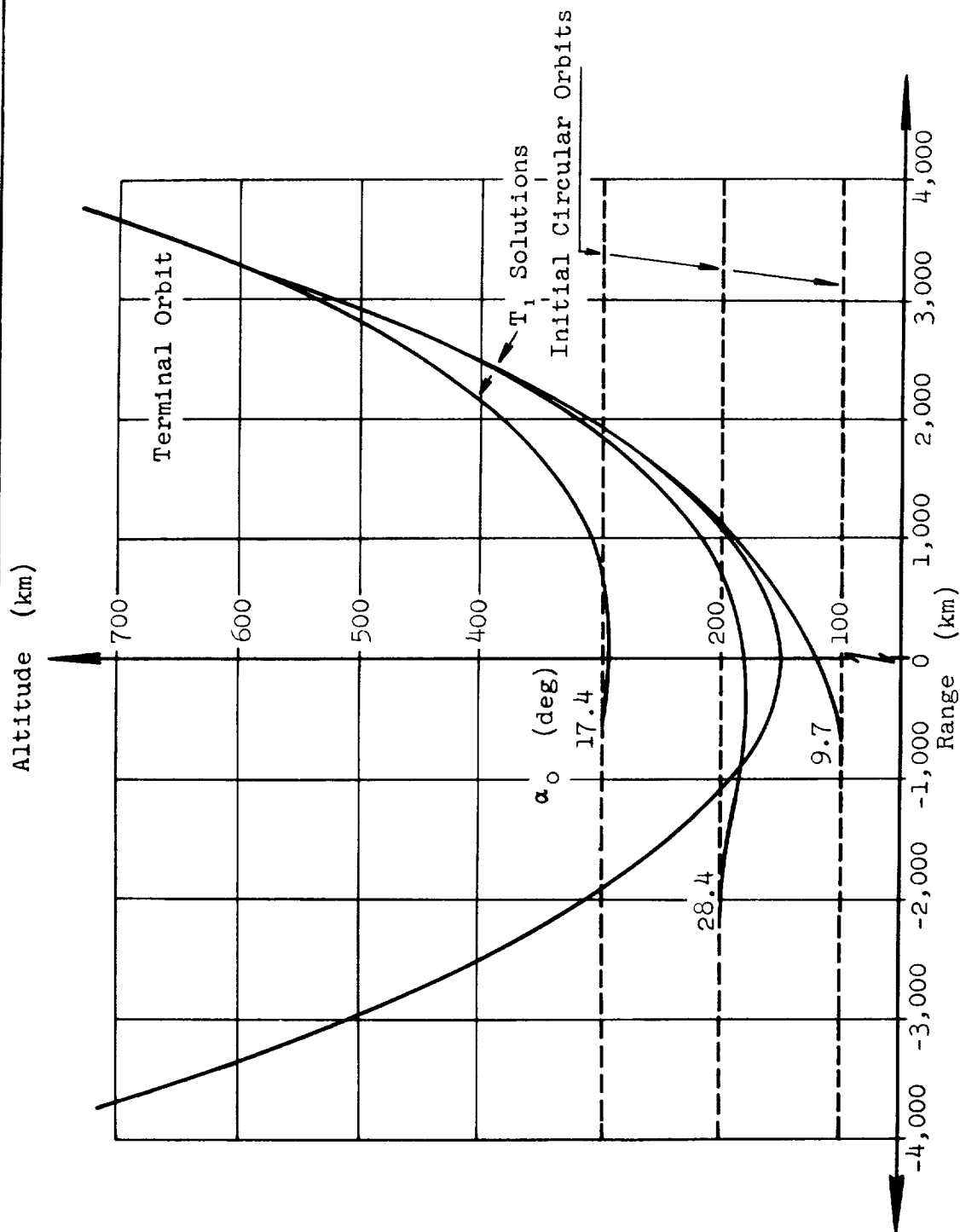
FIG. 38. OPTIMUM TRAJECTORIES FOR T_1 SOLUTIONS
FOR $F/W_o = .5$; $I_{sp} = 1,500$ SEC
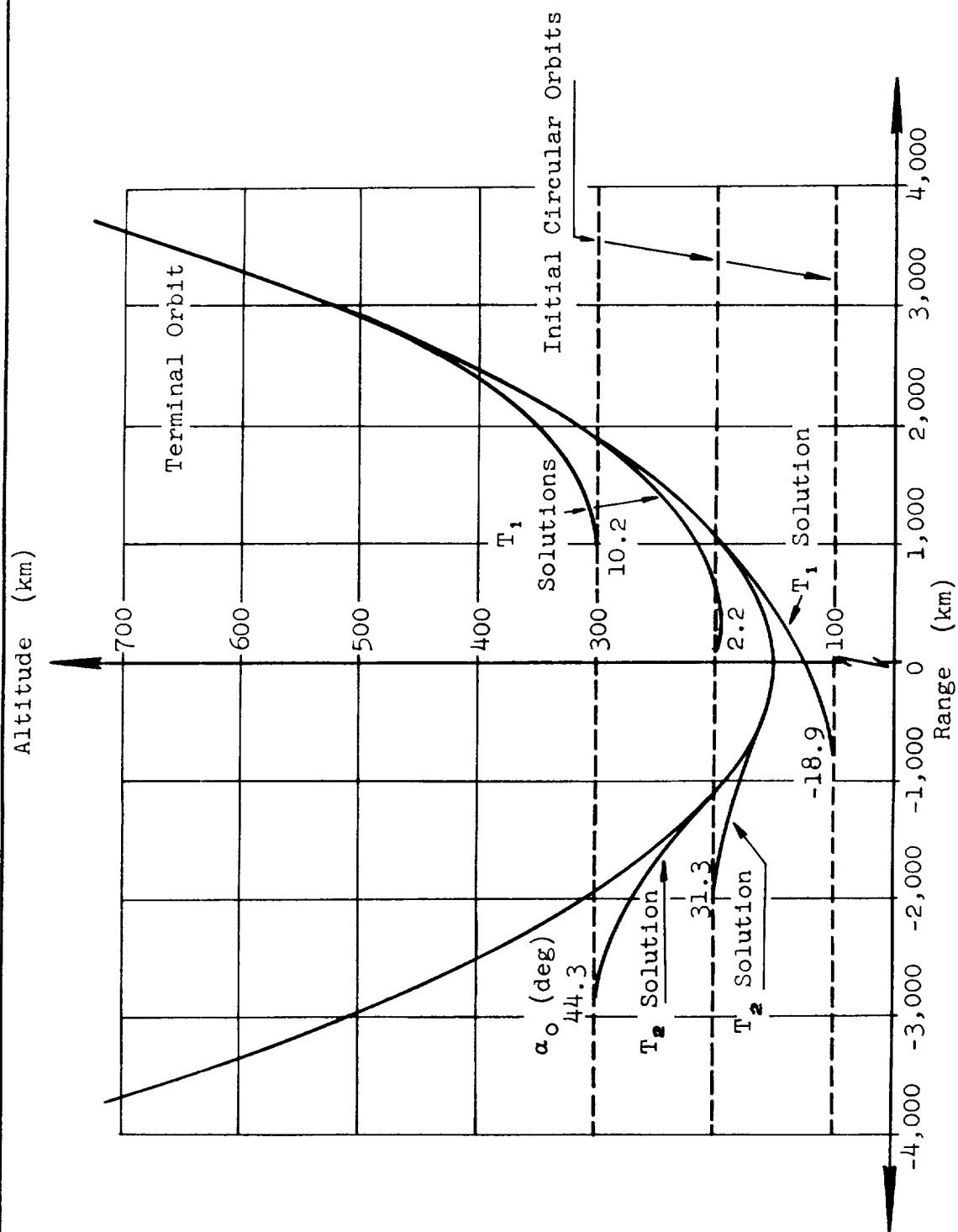
FIG. 39. OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS
FOR $F/W_0 = 1$; $I_{sp} = 400$ SEC
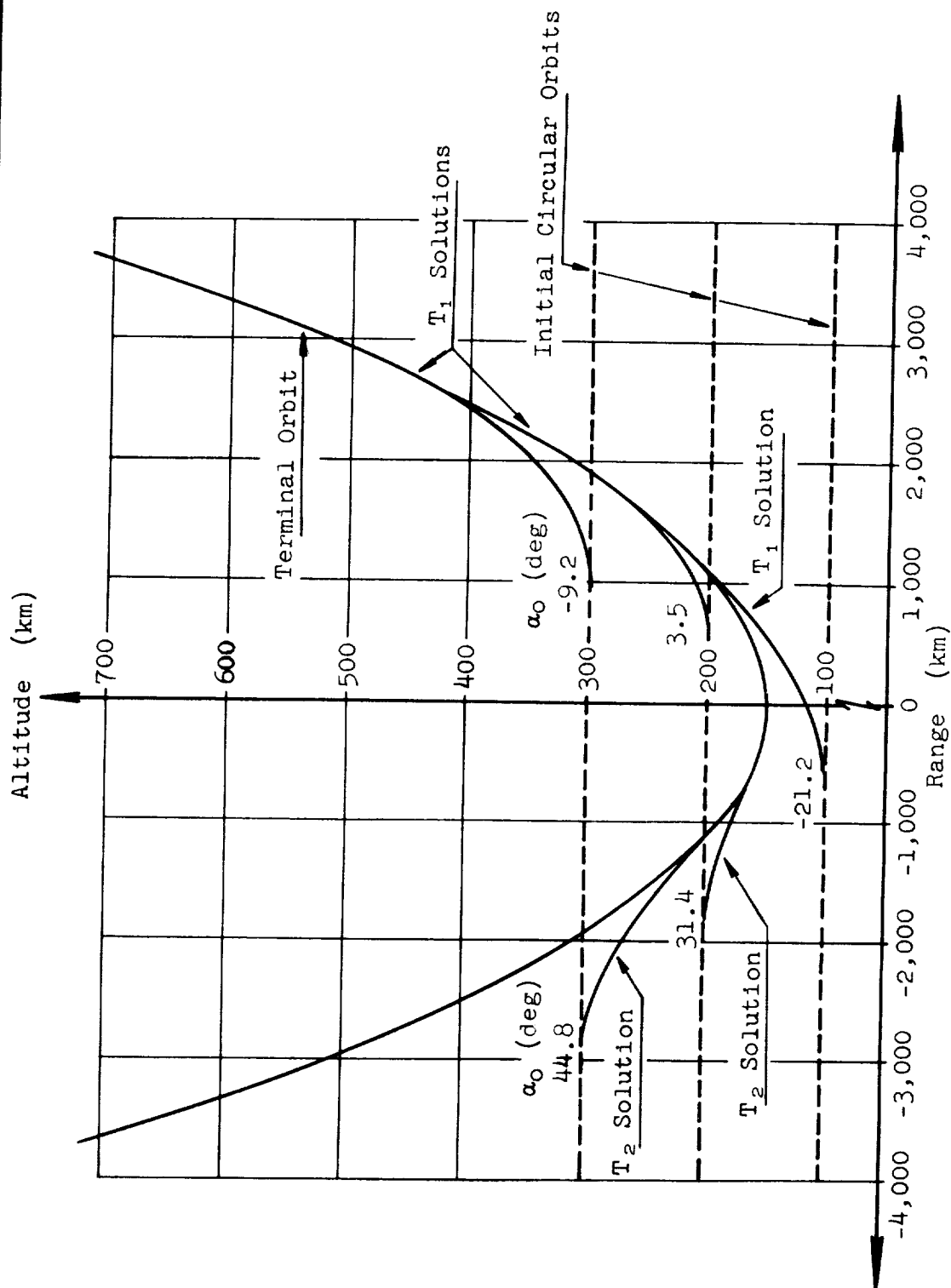
FIG. 40. OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS
FOR F/W₀ = 1; Isp = 500 SEC
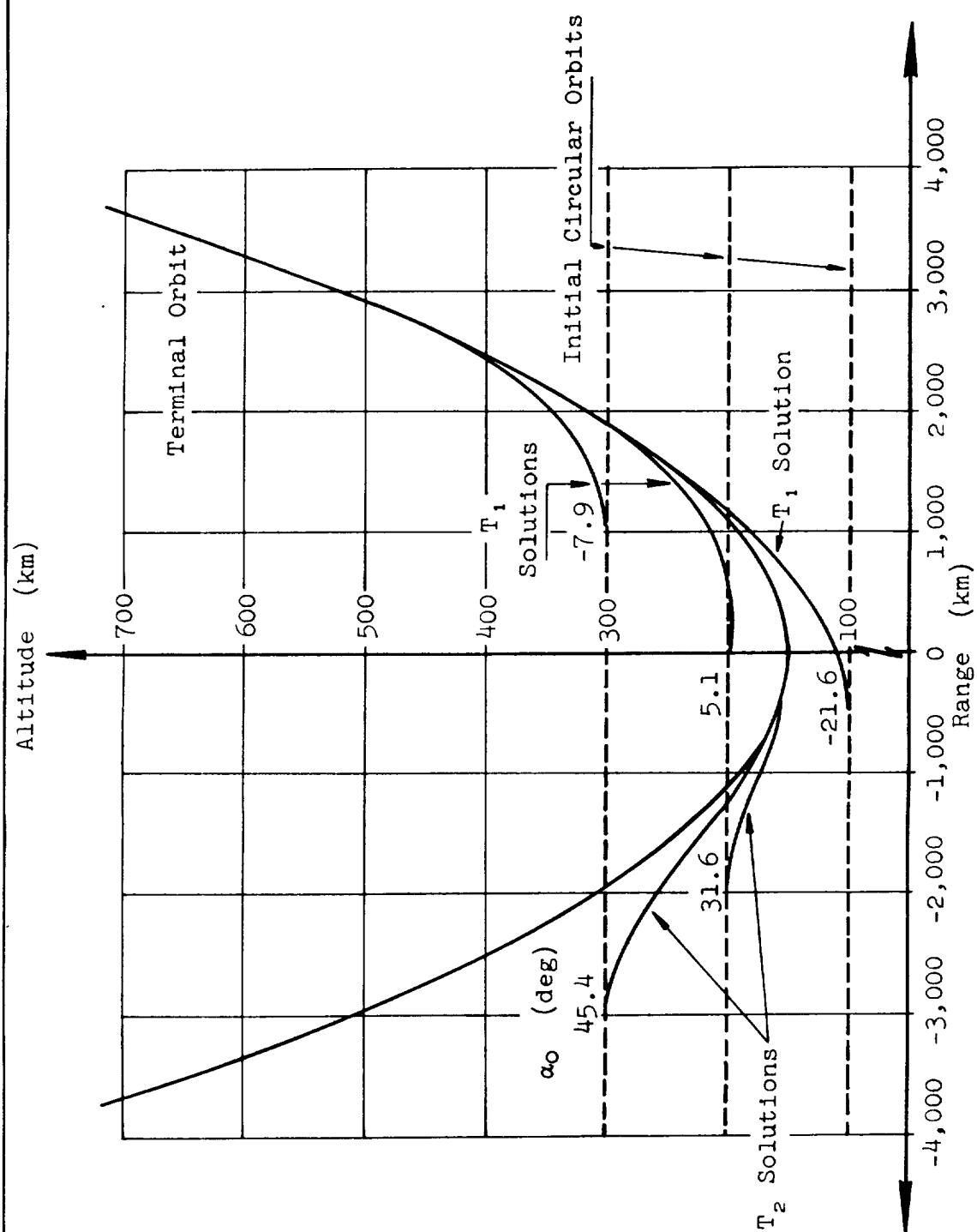
FIG. 41. OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS FOR $F/W_o = 1$; $I_{sp} = 700$ SEC
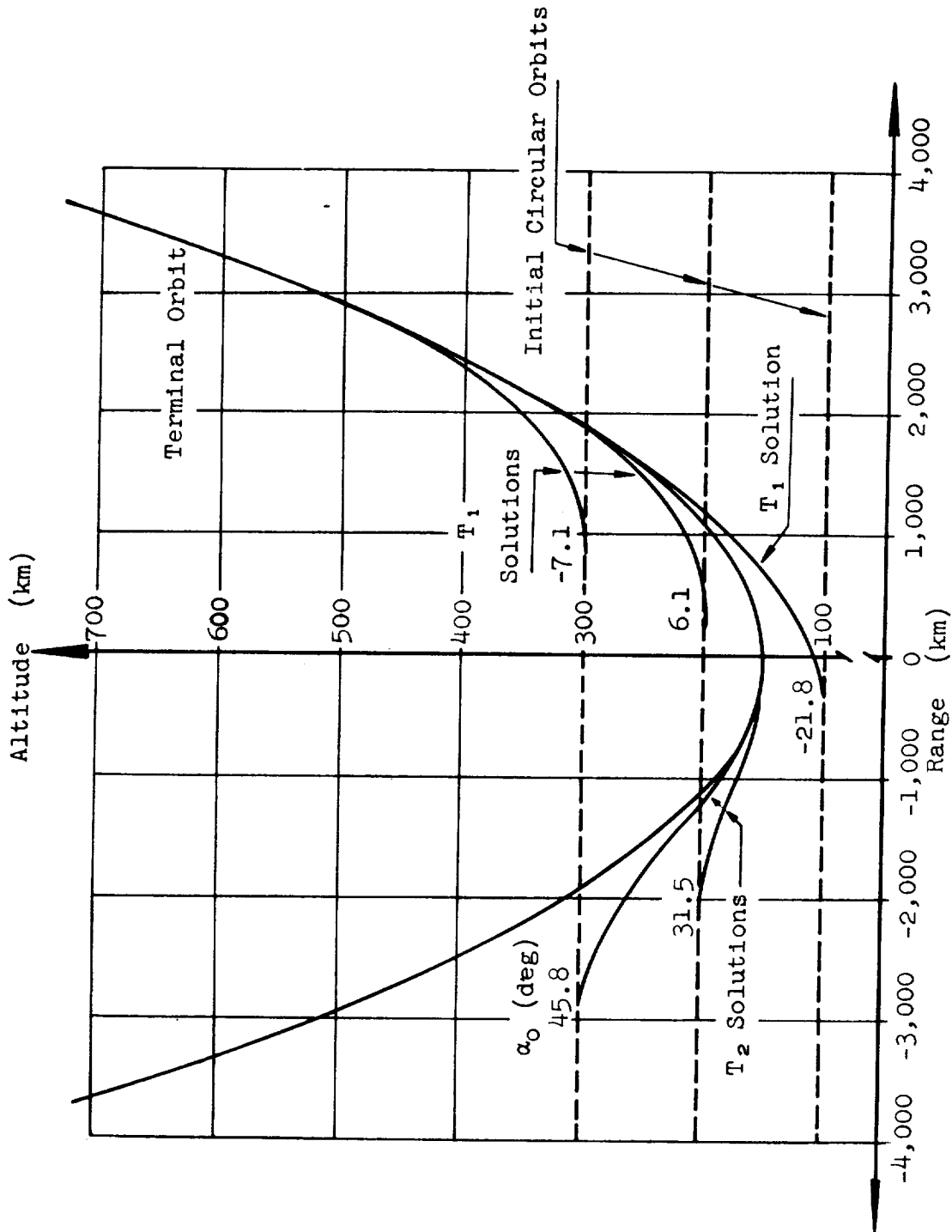
FIG. 42. OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS
FOR $F/W_o = 1$; $I_{sp} = 900$ SEC
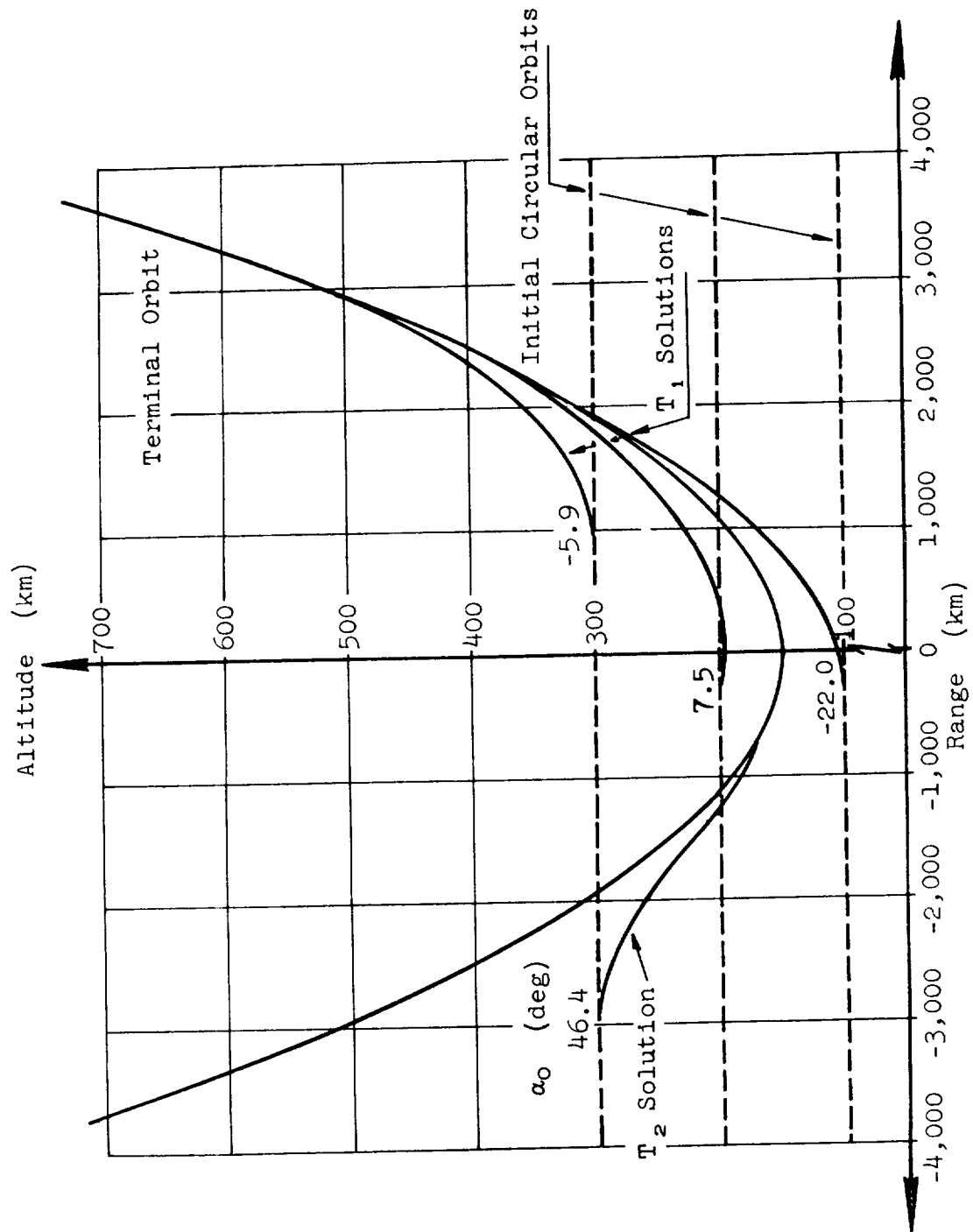
FIG. 43. OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS

FOR $F/W_o = 1$; $I_{sp} = 1,500$ SEC
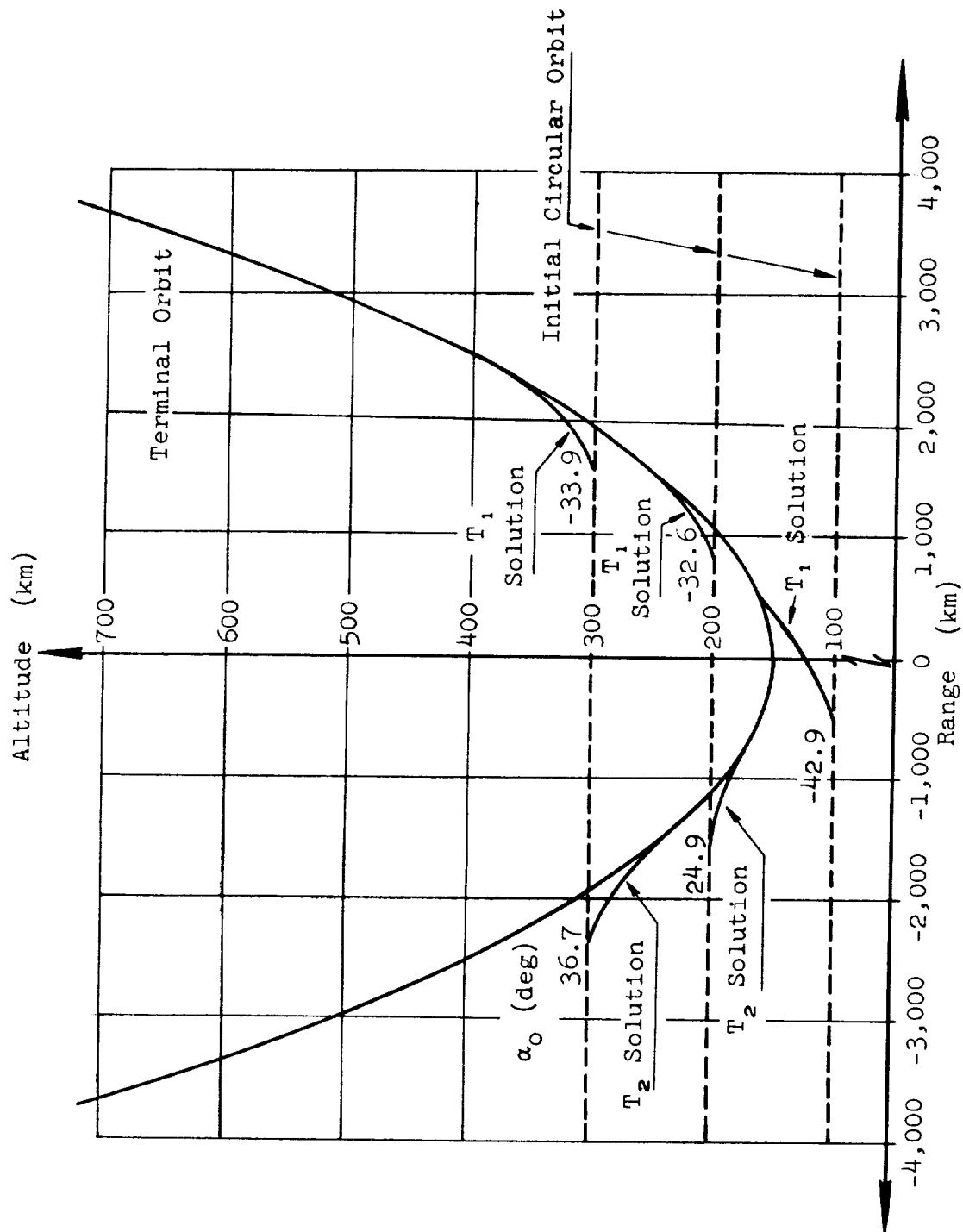
FIG. 44. OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS
FOR $F/W_o = 2$; $I_{sp} = 400$ SEC
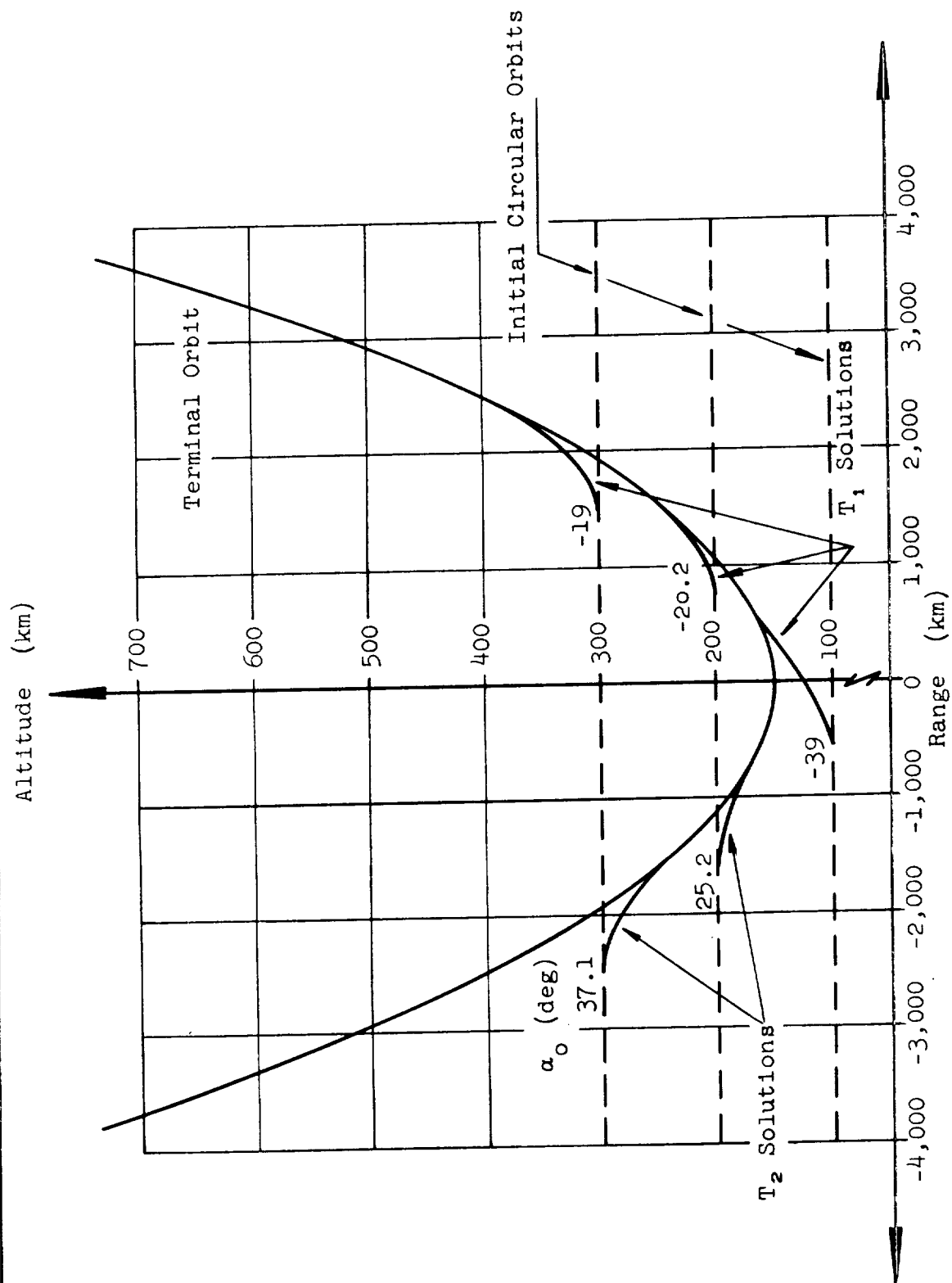
FIG. 45. OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS
FOR F/Wo = 2; Isp = 500 SEC
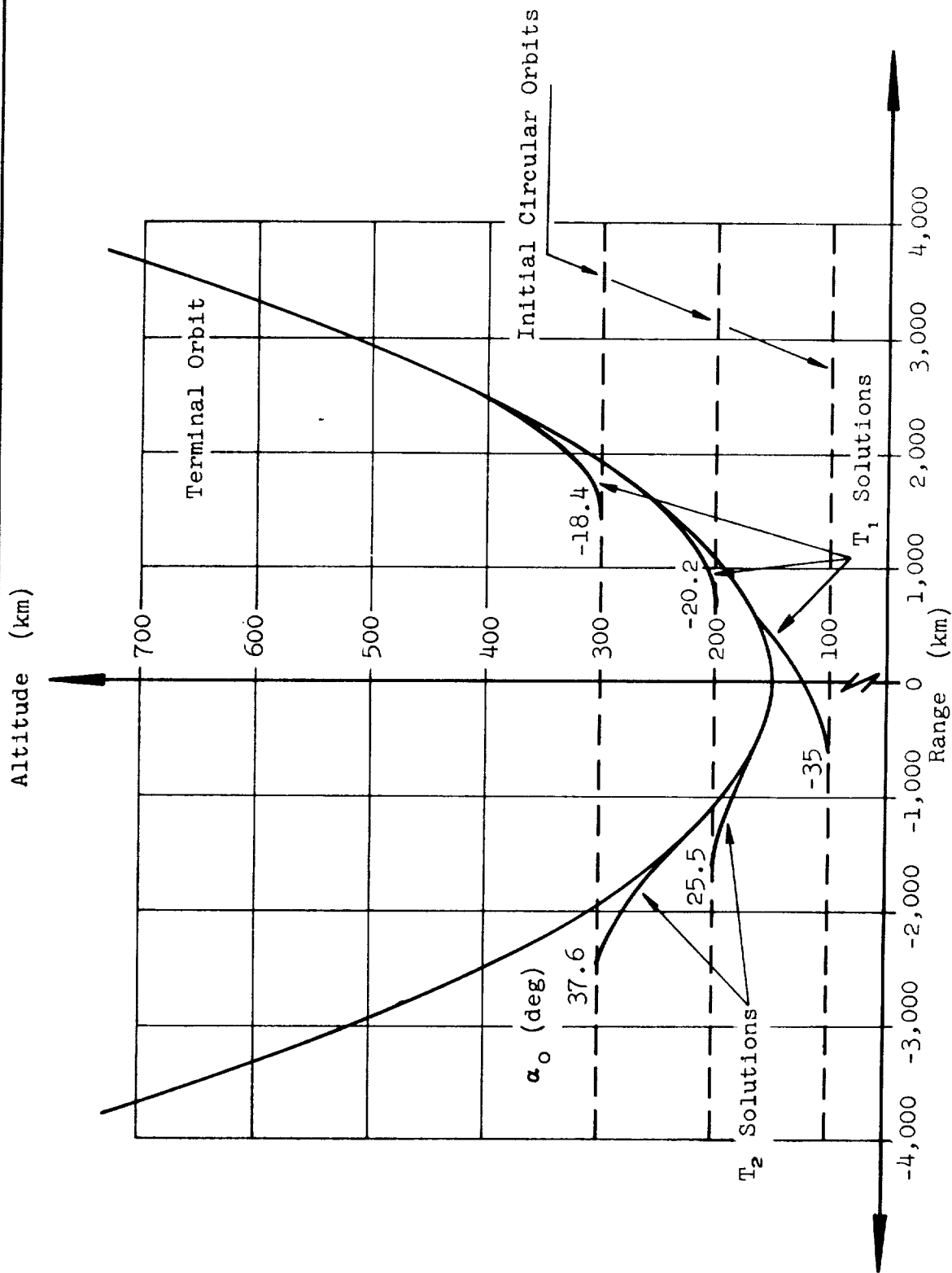
FIG. 46. OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS
FOR $F/W_o = 2$; $I_{sp} = 700$ SEC
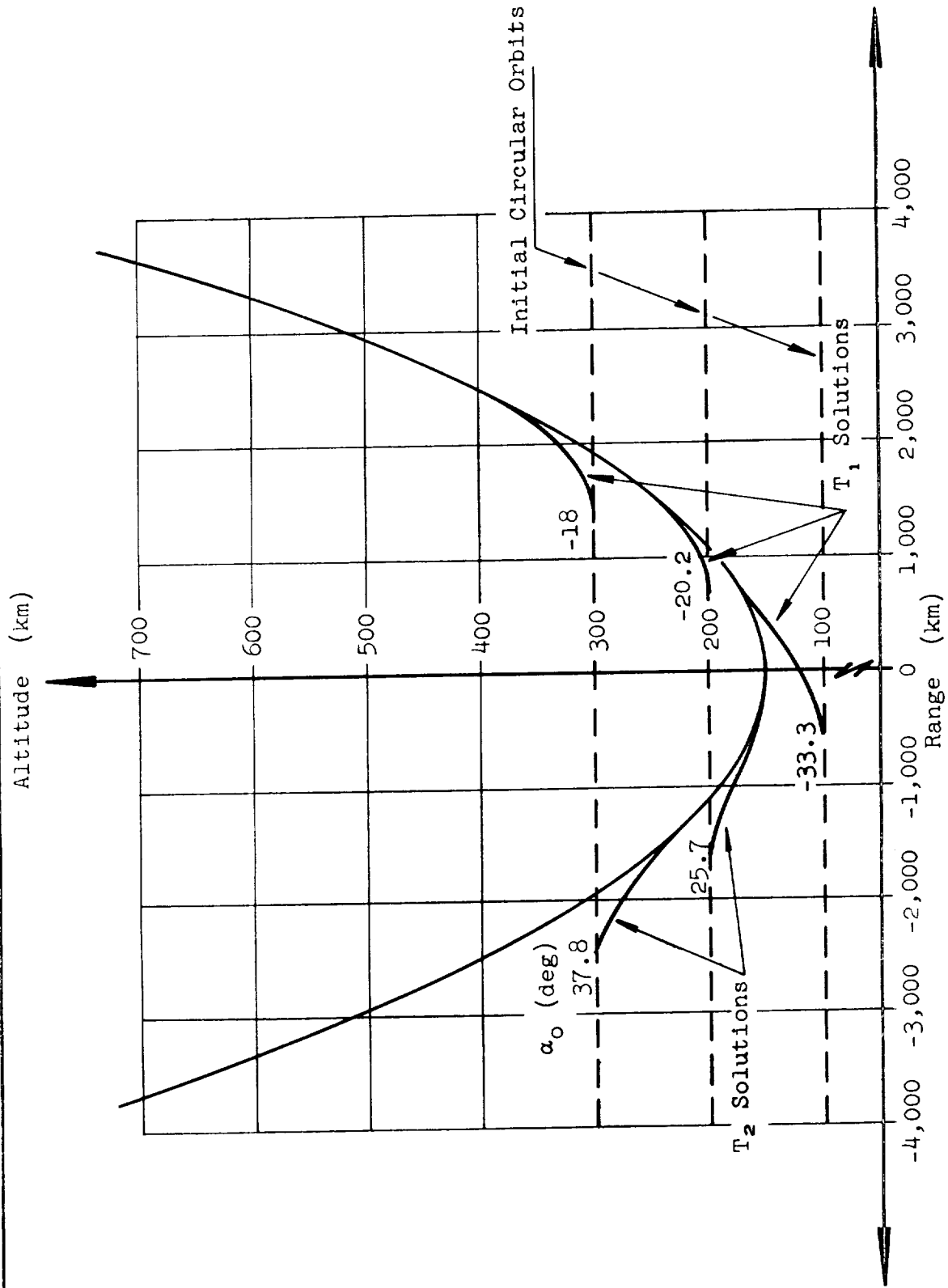
FIG. 47. OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS
FOR $F/W_o = 2$; $I_{sp} = 900$ SEC

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FIG. 48. OPTIMUM TRAJECTORIES FOR BOTH TYPES OF SOLUTIONS
FOR $F/W_0 = 2$; $I_{sp} = 1,500$ SEC
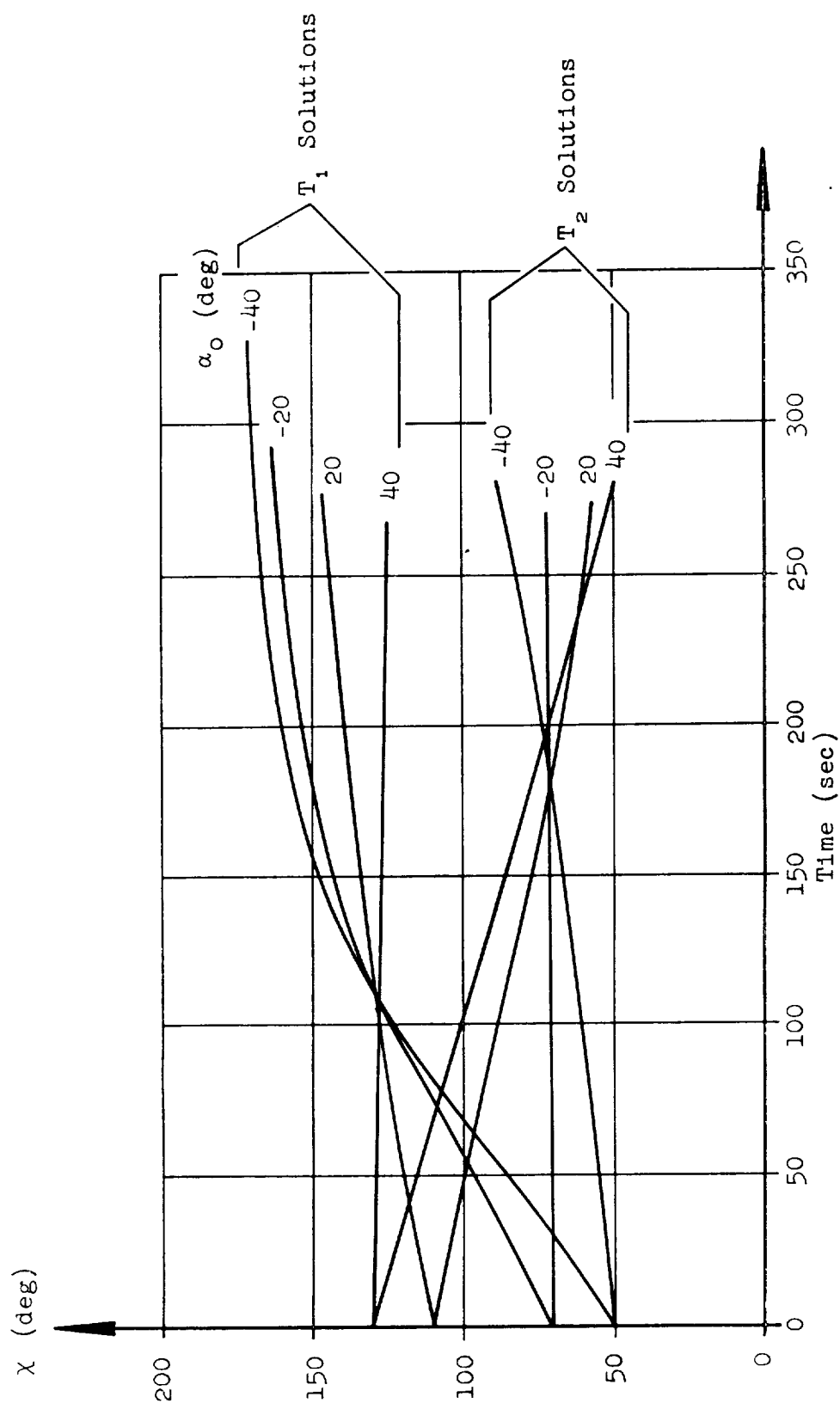
FIG. 49. NON-OPTIMUM CONTROL HISTORIES FOR PARAMETERS OF α_o

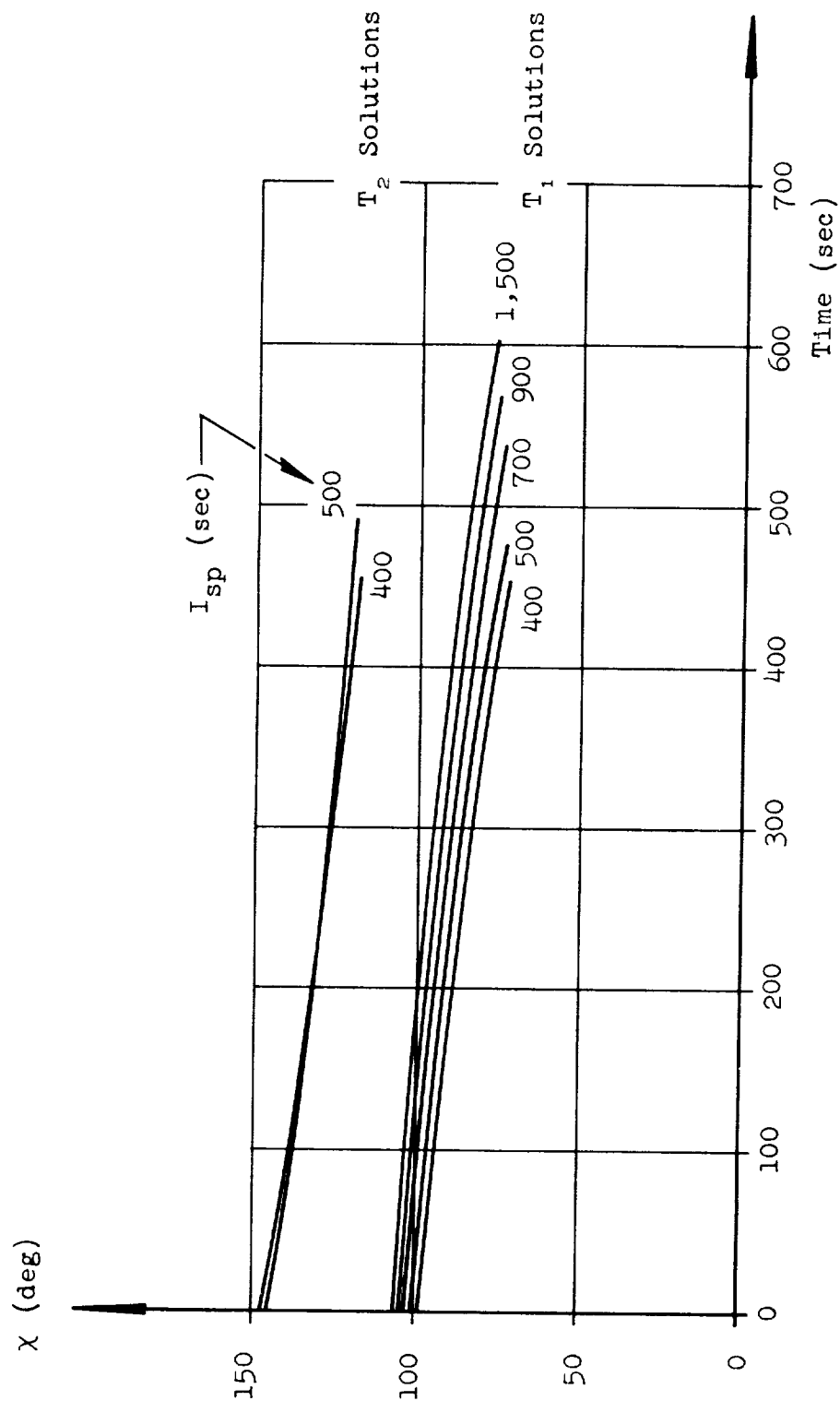FOR $F/W_o = 1$; $I_{sp} = 700$ sec; $y_o = 300$ km
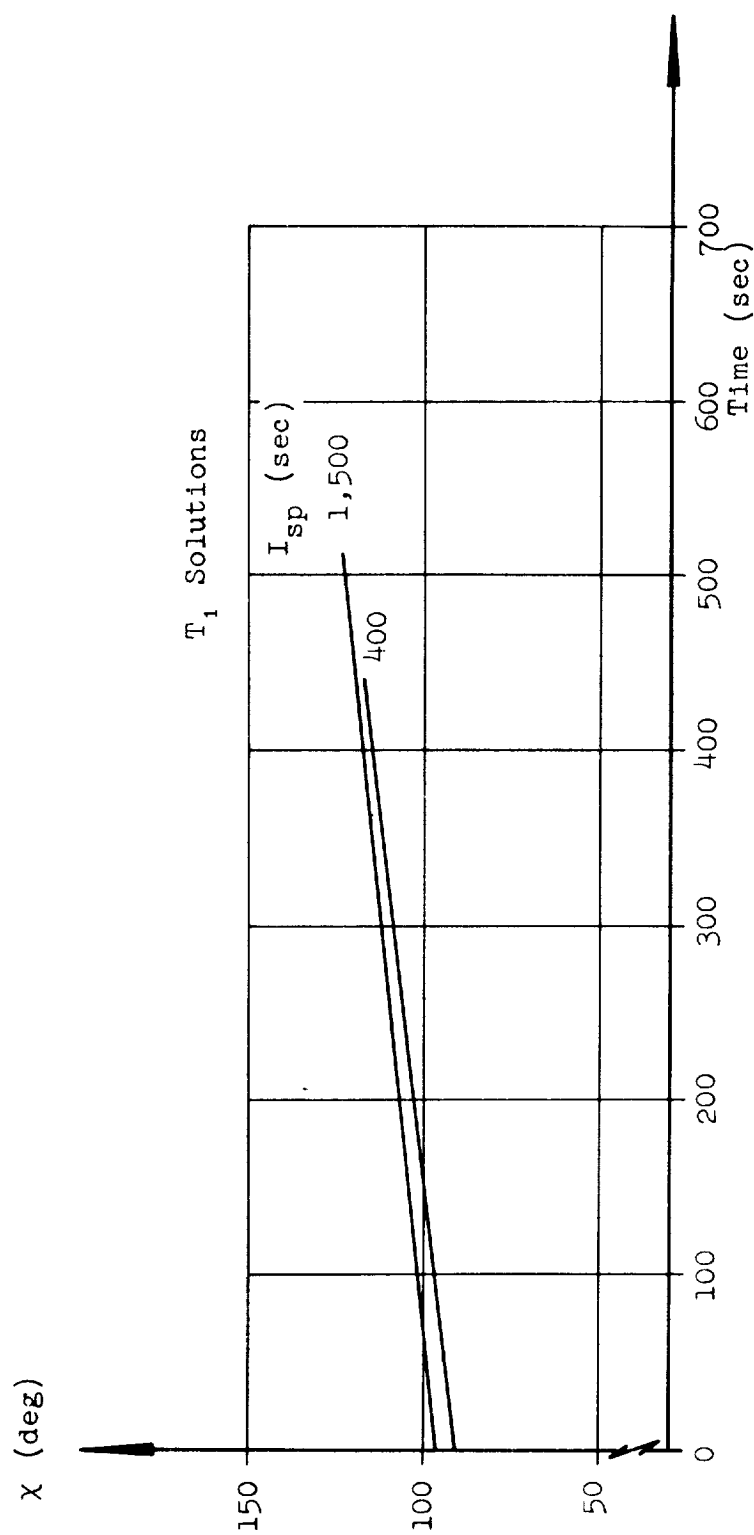
FIG. 50.   OPTIMUM CONTROL HISTORIES FOR PARAMETERS OF I_{sp}
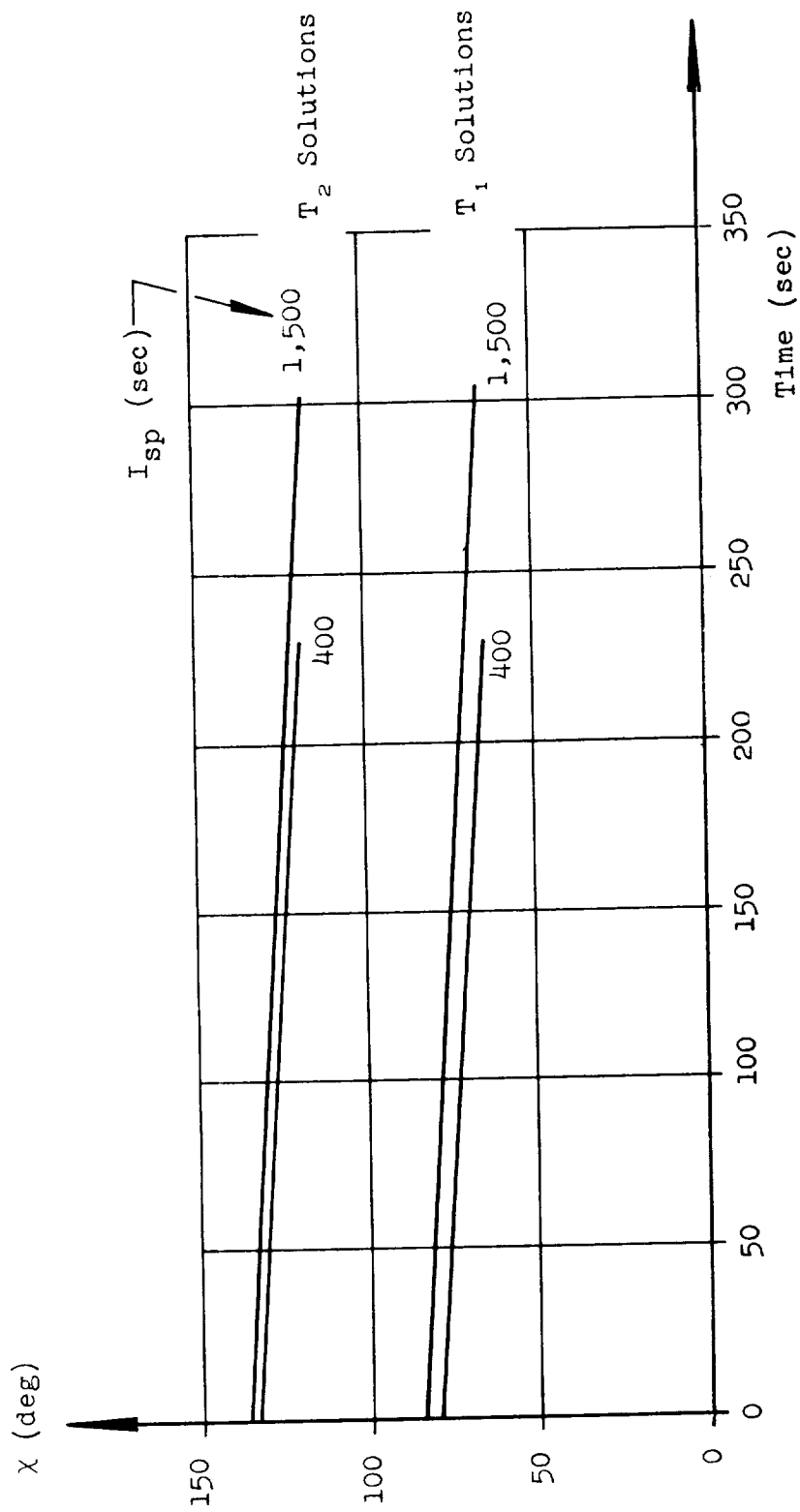
FOR $F/W_o = .5$; $y_o = 100$ km
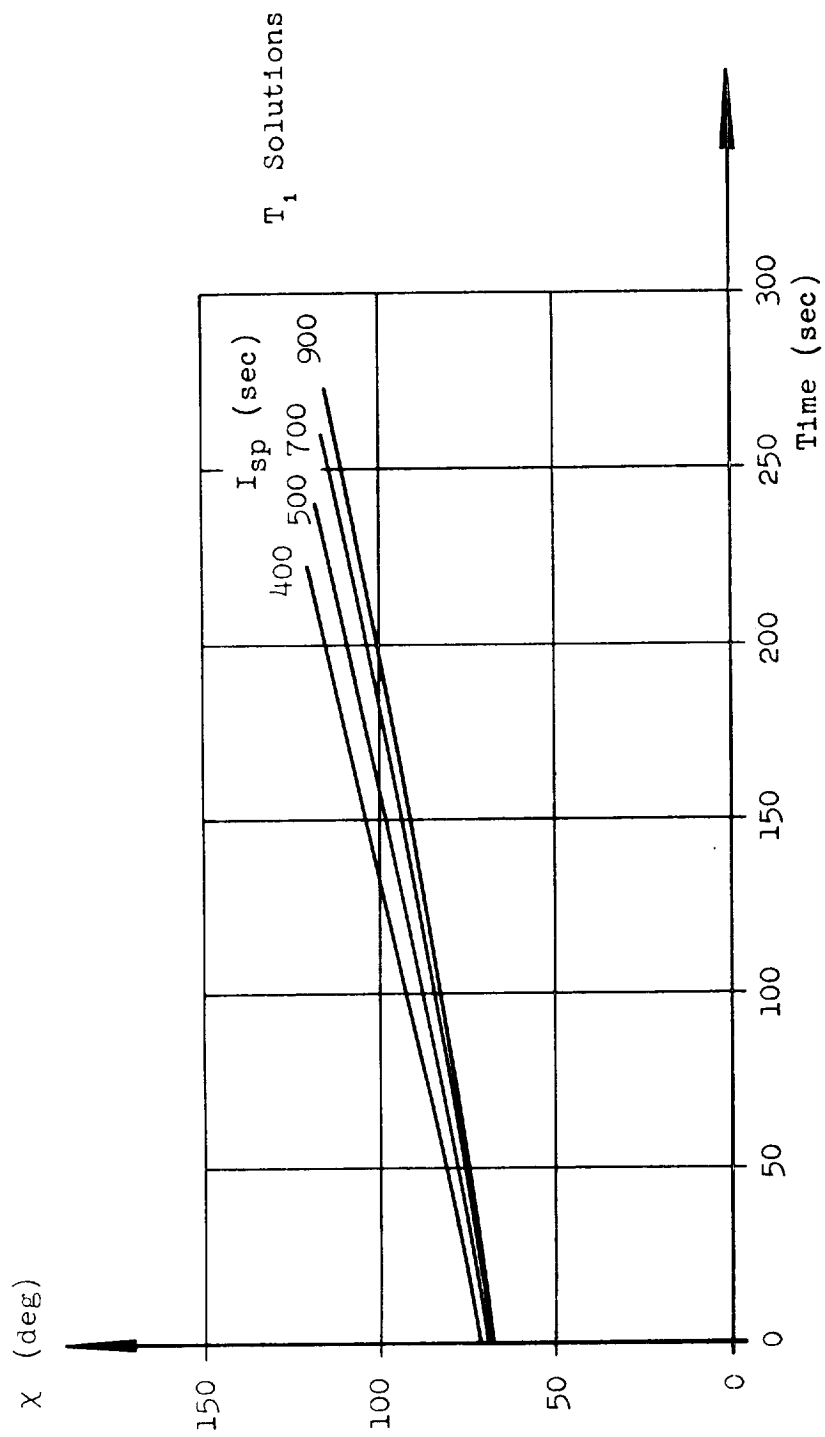
FIG. 51. OPTIMUM CONTROL HISTORIES FOR PARAMETERS OF I_{sp}
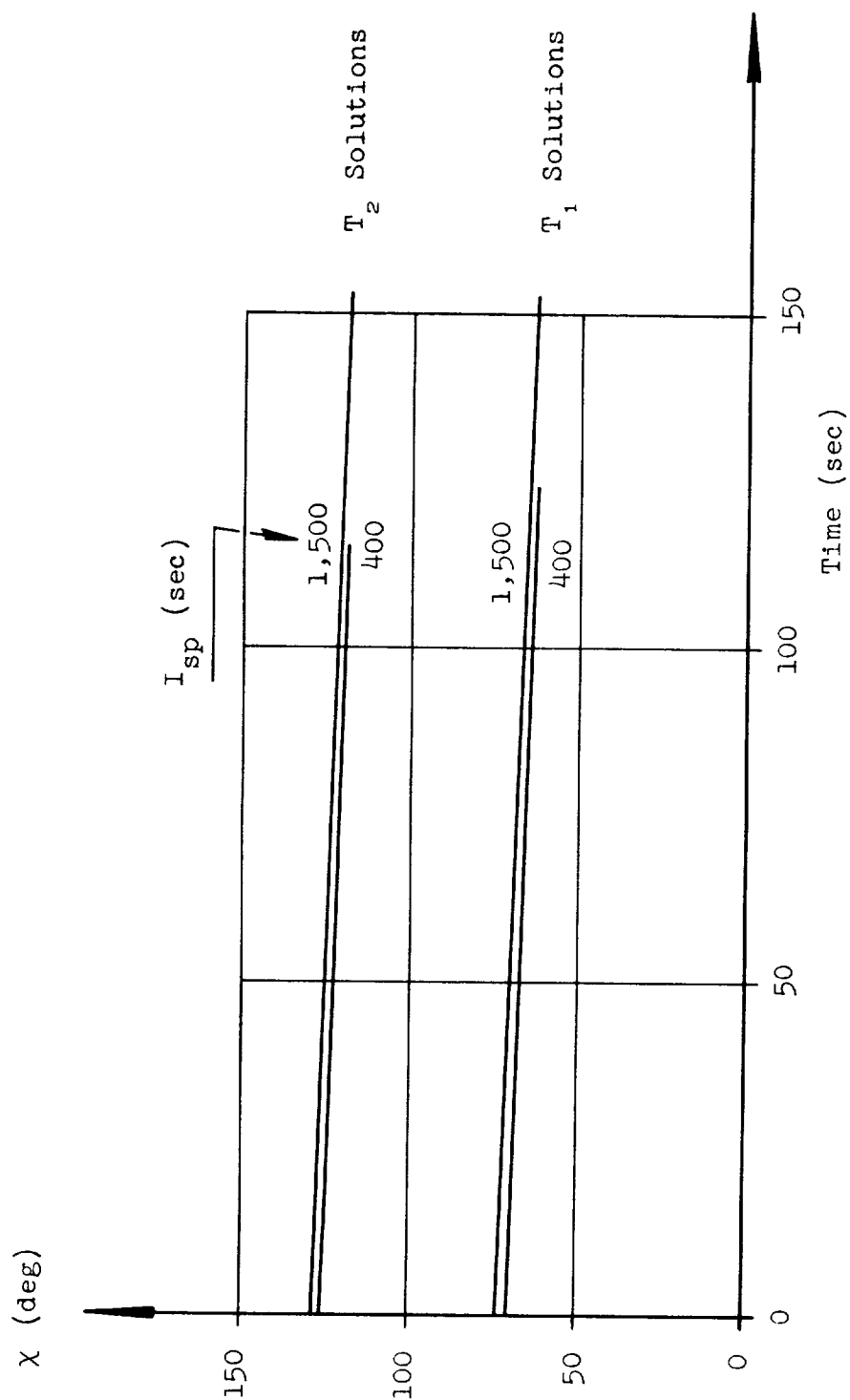
FOR $F/W_O = 1$; $y_O = 100$ km
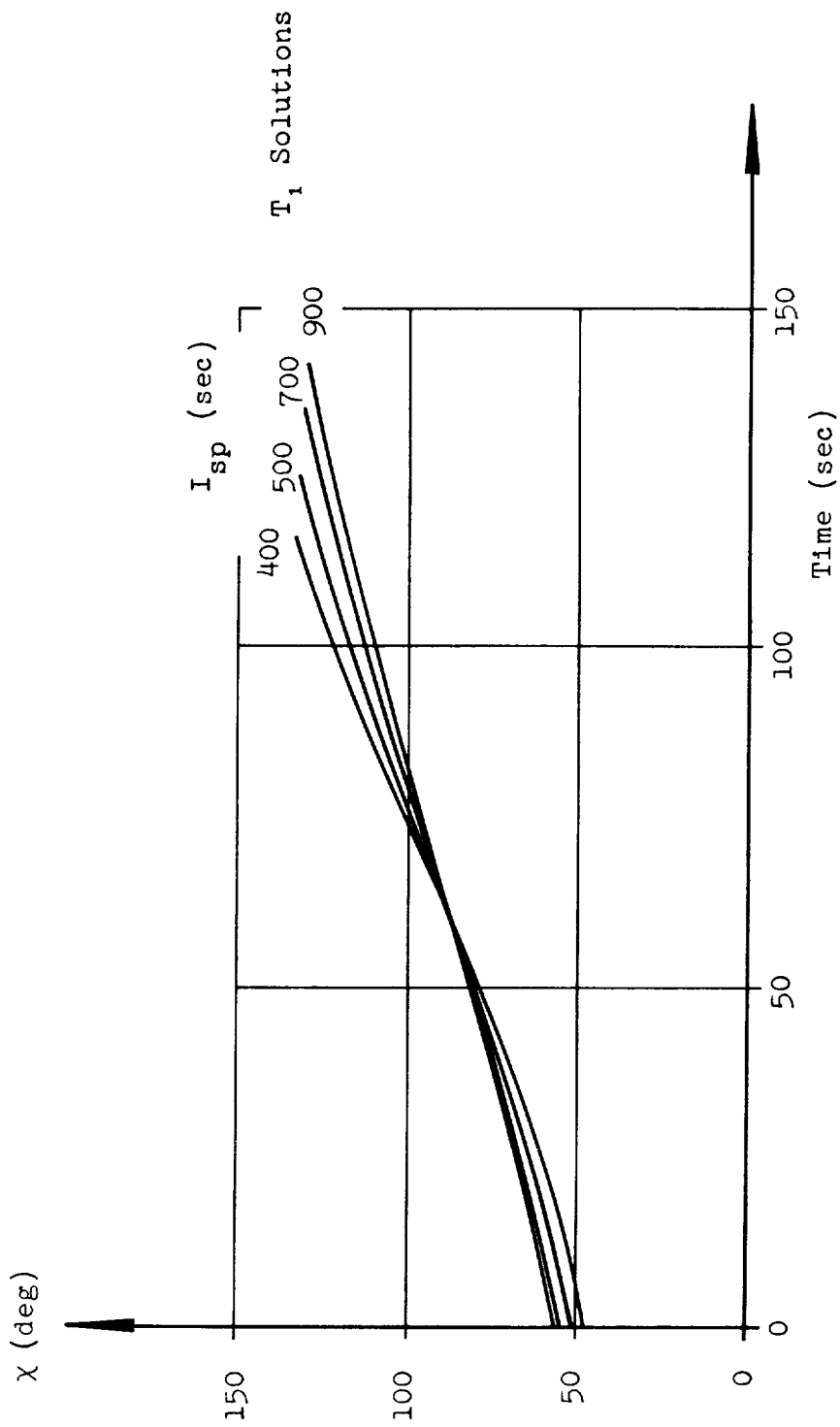
FIG. 52. OPTIMUM CONTROL HISTORIES FOR PARAMETERS OF I_{sp}

FOR $F/W_o = 2$; $y_o = 100$ km

FIG. 53. OPTIMUM CONTROL HISTORIES FOR PARAMETERS OF I_{sp}

FOR $F/W_o = .5$; $y_o = 200$ km
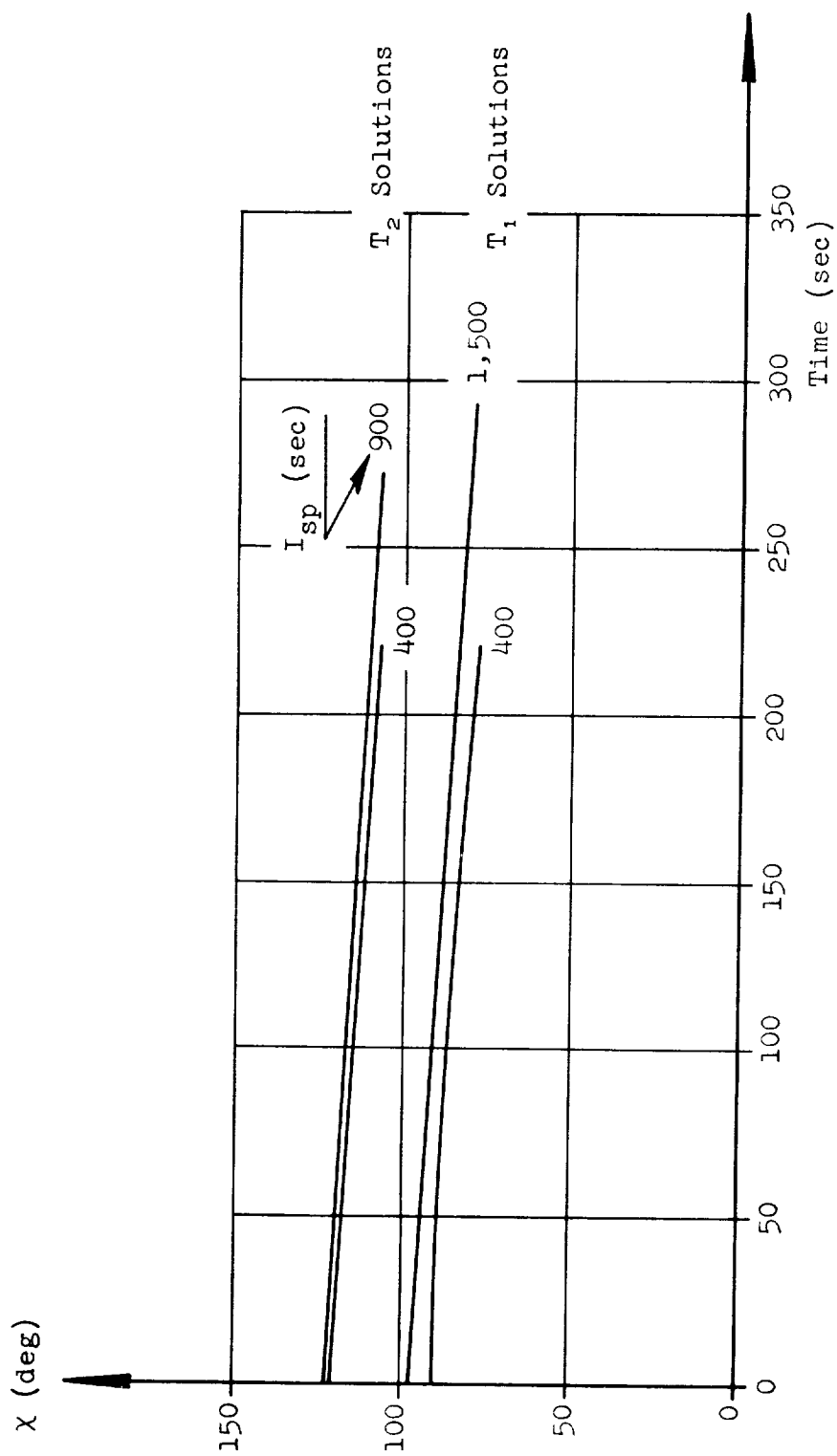
FIG. 54. OPTIMUM CONTROL HISTORIES FOR PARAMETERS OF I_{sp}

FOR $F/W_o = 1$; $y_o = 200$ km
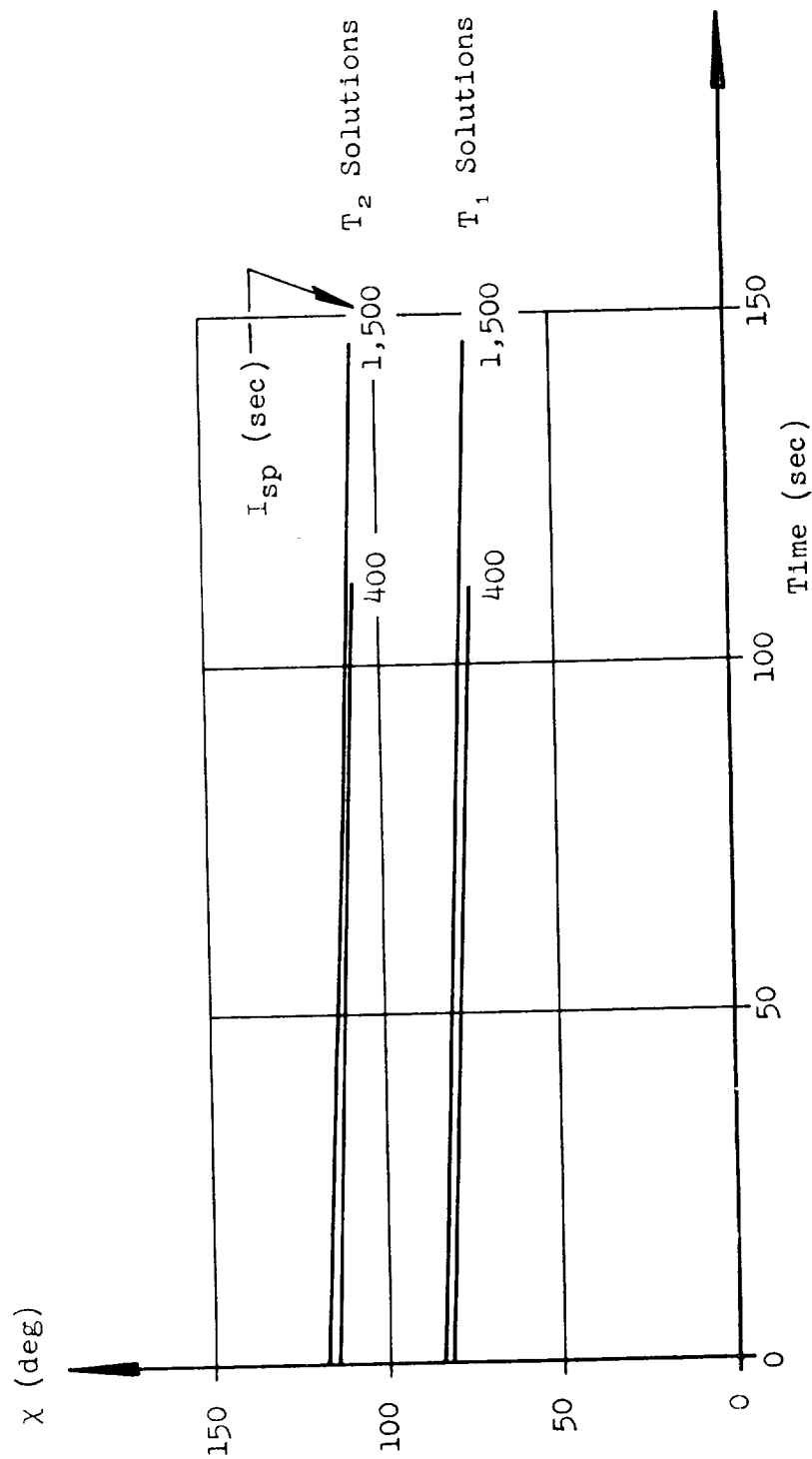
FIG. 55. OPTIMUM CONTROL HISTORIES FOR PARAMETERS OF I_{sp}
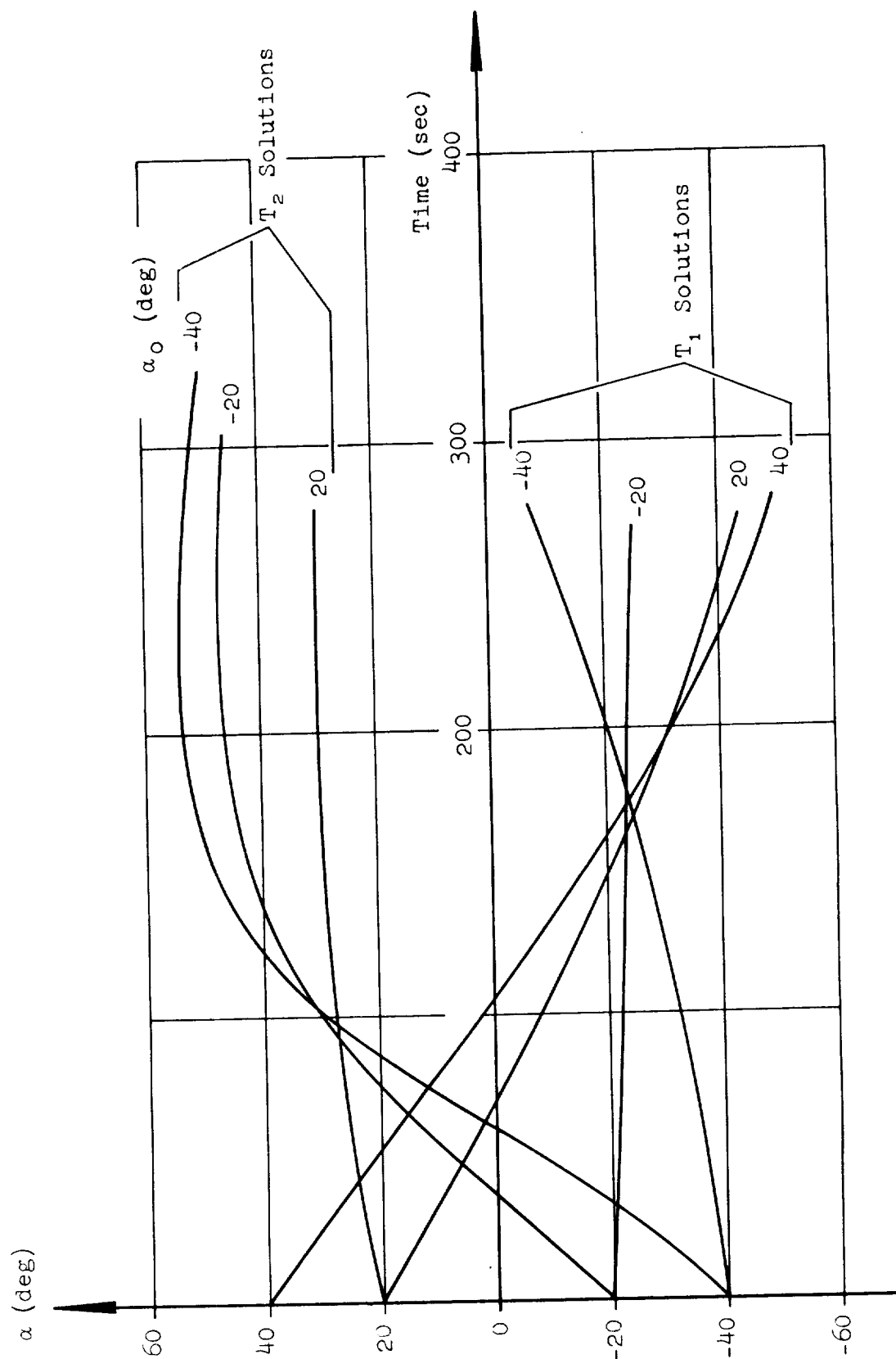
FOR $F/W_o = 2$; $y_o = 200$ km

FIG. 56. OPTIMUM CONTROL HISTORIES FOR PARAMETERS OF I_{sp}

FOR $F/W_o = .5$; $y_o = 300$ km

FIG. 57. OPTIMUM CONTROL HISTORIES FOR PARAMETERS OF I_{sp}

FOR $F/W_o = 1$; $y_o = 300$ km

FIG. 58. OPTIMUM CONTROL HISTORIES FOR PARAMETERS OF I_{sp}

FOR $F/W_O = 2$; $y_O = 300$ km

FIG. 59. NON-OPTIMUM ANGLE OF ATTACK HISTORIES
FOR PARAMETERS OF α_o
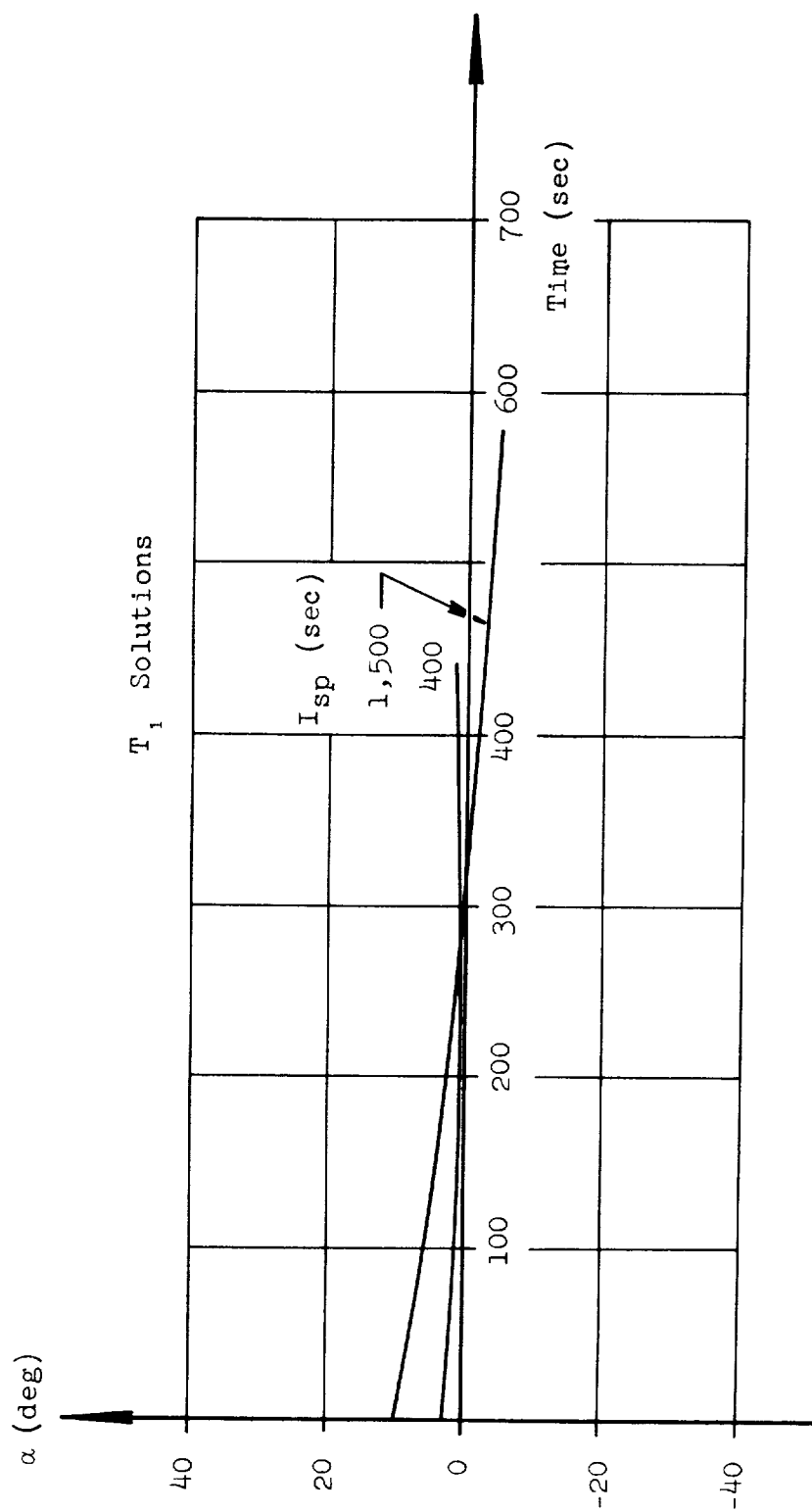
FOR $F/W_o = 1$; $I_{sp} = 700$ sec; $y_o = 300$ km

FIG. 60.   OPTIMUM ANGLE OF ATTACK HISTORIES FOR PARAMETERS OF I_{sp}
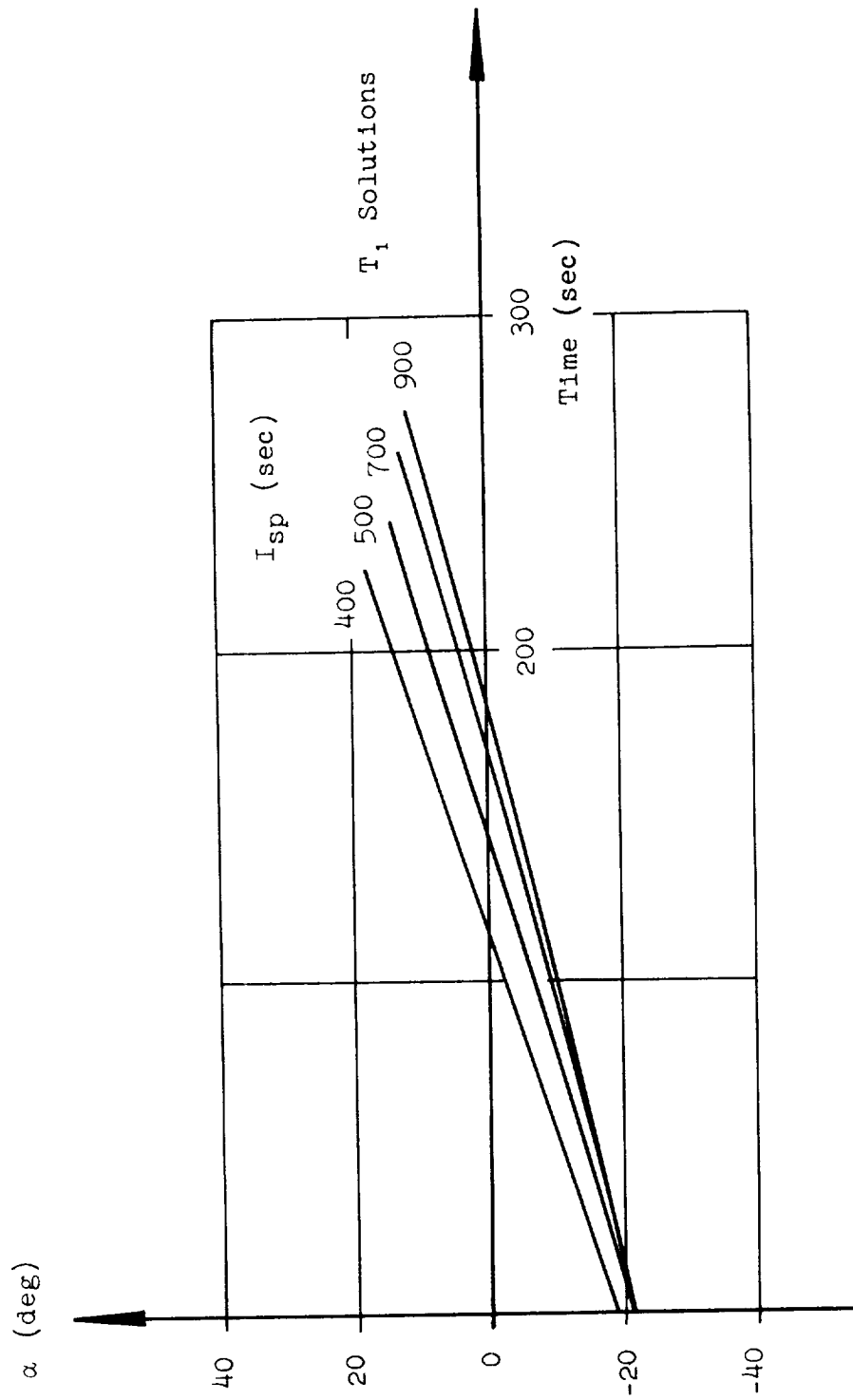
FOR $F/W_o = .5$; $y_o = 100$ km

FIG. 61. OPTIMUM ANGLE OF ATTACK HISTORIES FOR PARAMETERS OF I_{sp}

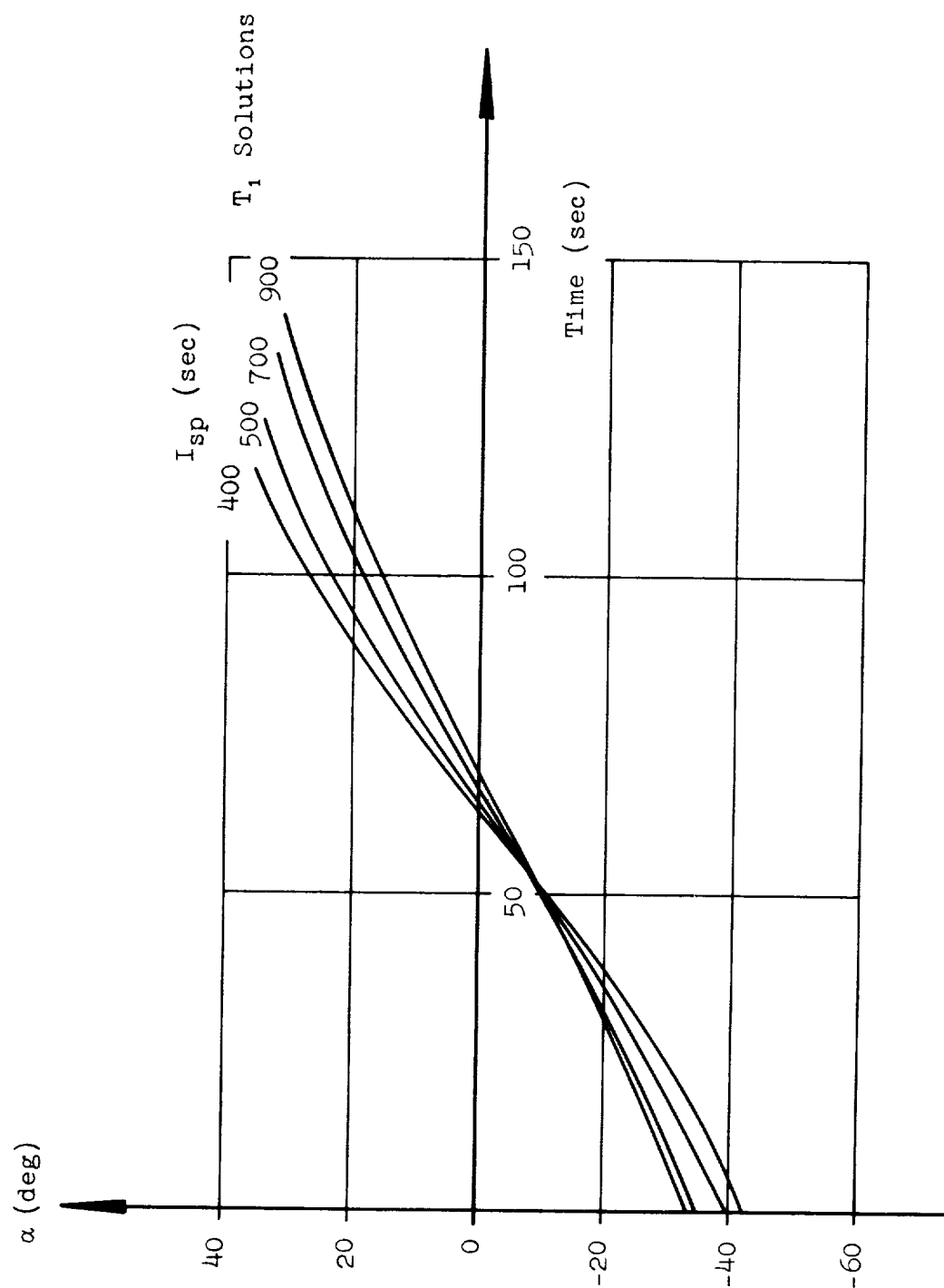FOR $F/W_o = 1$; $y_o = 100$ km

FIG. 62.   OPTIMUM ANGLE OF ATTACK HISTORIES FOR PARAMETERS OF I_{sp}
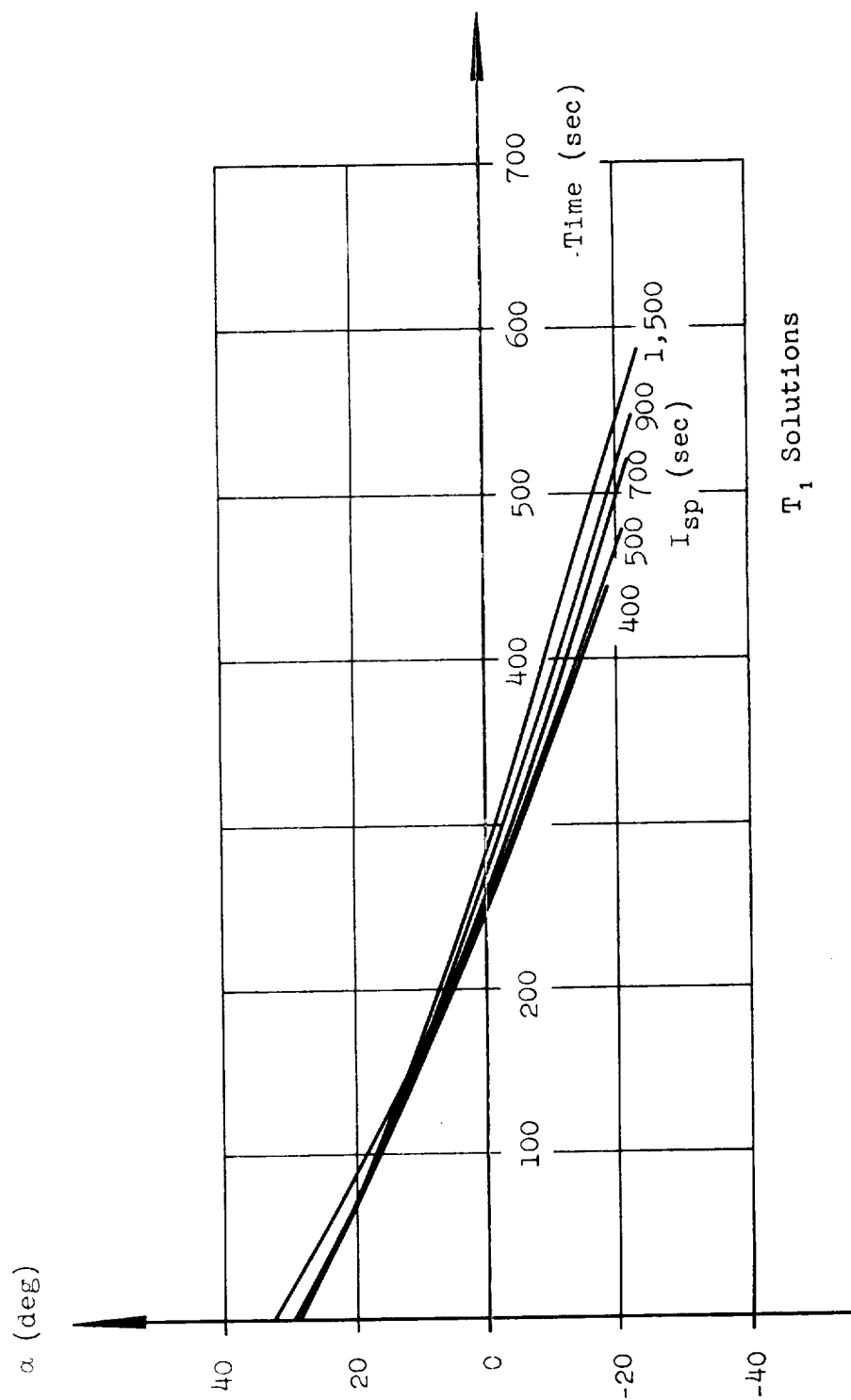
FOR $F/W_0 = 2$; $y_0 = 100$ km

FIG. 63.   OPTIMUM ANGLE OF ATTACK HISTORIES FOR PARAMETERS OF I_{sp}
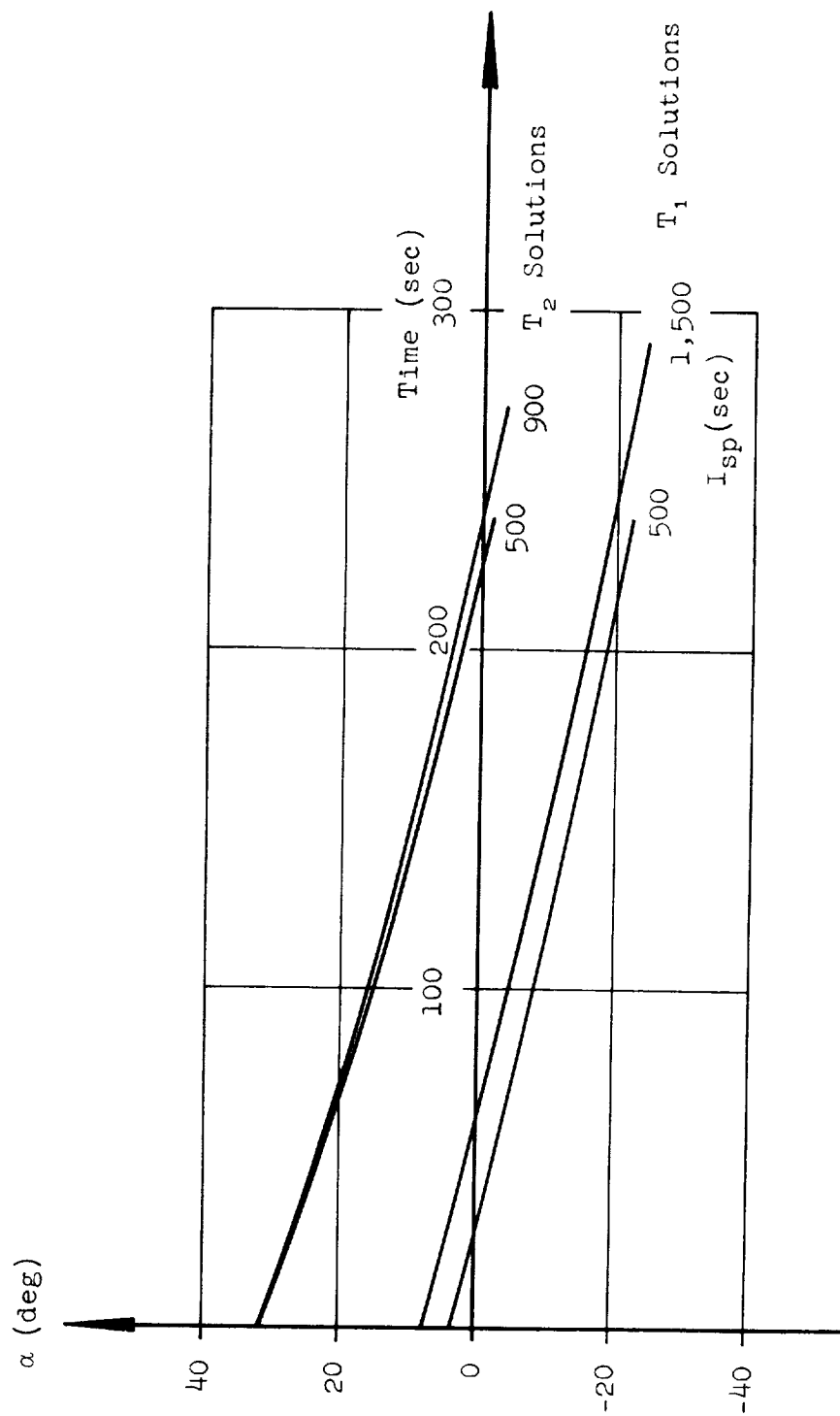
FOR $F/W_o = .5$; $y_o = 200$ km

FIG. 64.   OPTIMUM ANGLE OF ATTACK HISTORIES FOR PARAMETERS OF I_{sp}

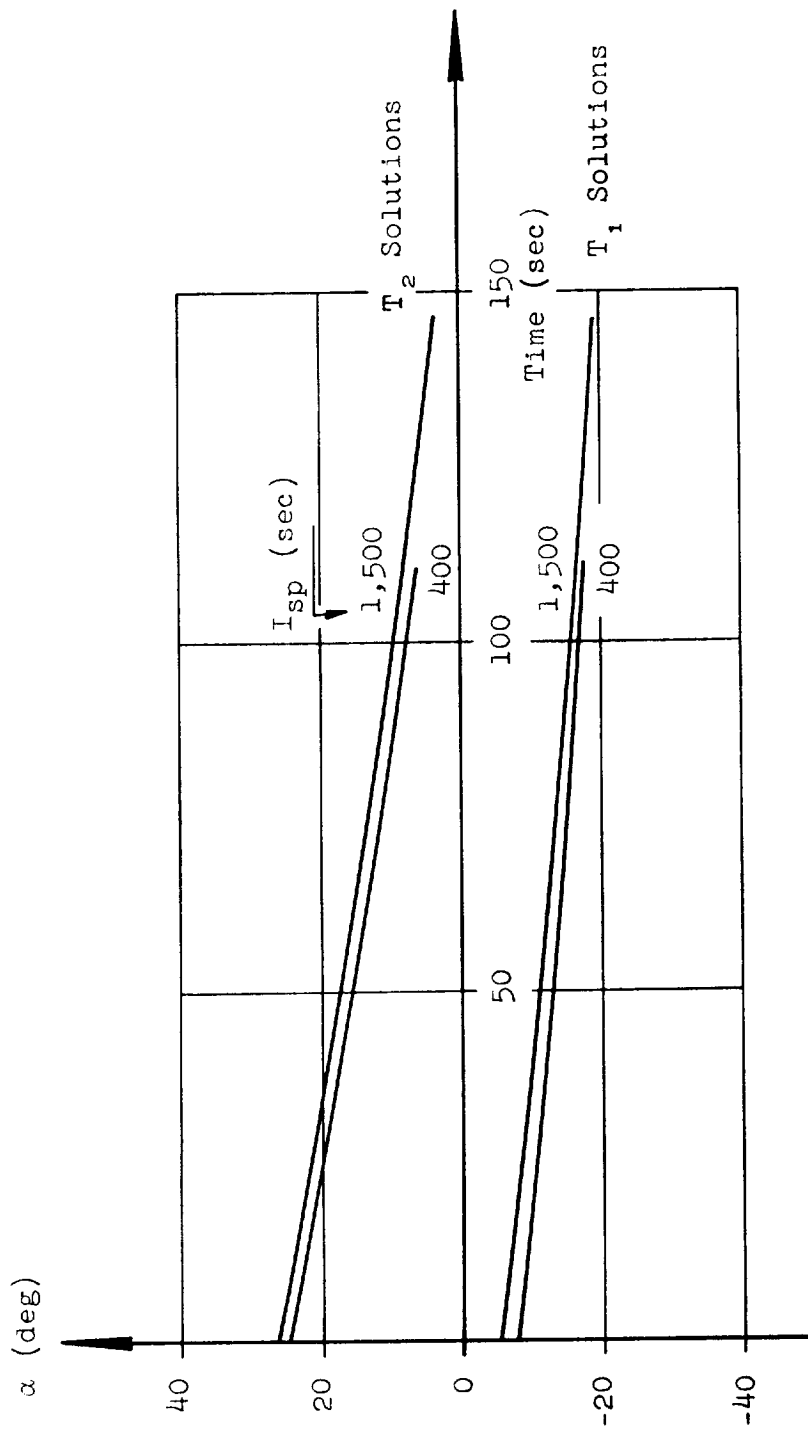FOR $F/W_o = 1$; $y_o = 200$ km

FIG. 65. OPTIMUM ANGLE OF ATTACK HISTORIES FOR PARAMETERS OF I_{sp}
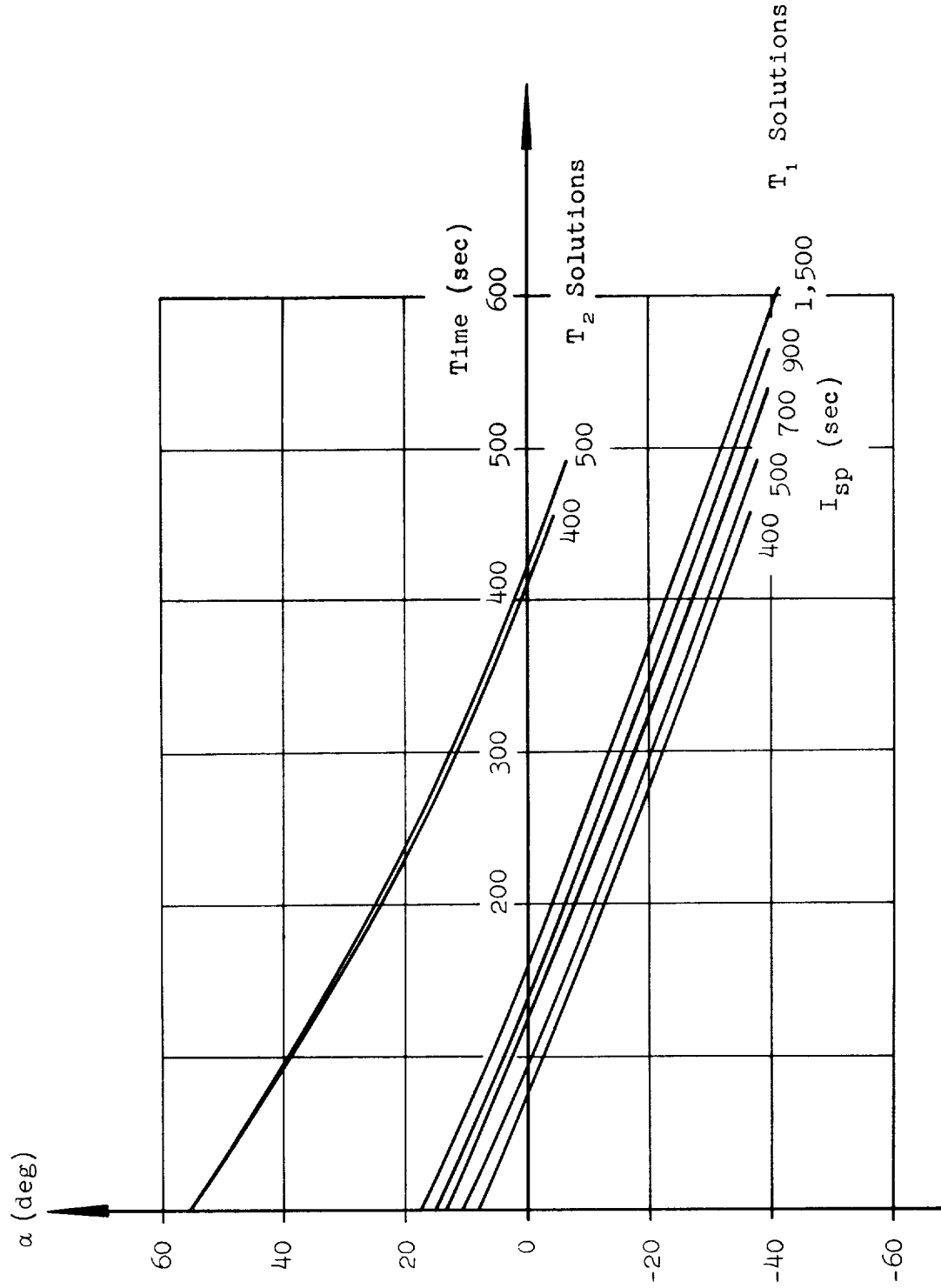
FOR $F/W_o = 2$; $y_o = 200$ km

FIG. 66.   OPTIMUM ANGLE OF ATTACK HISTORIES FOR PARAMETERS OF I_{sp}
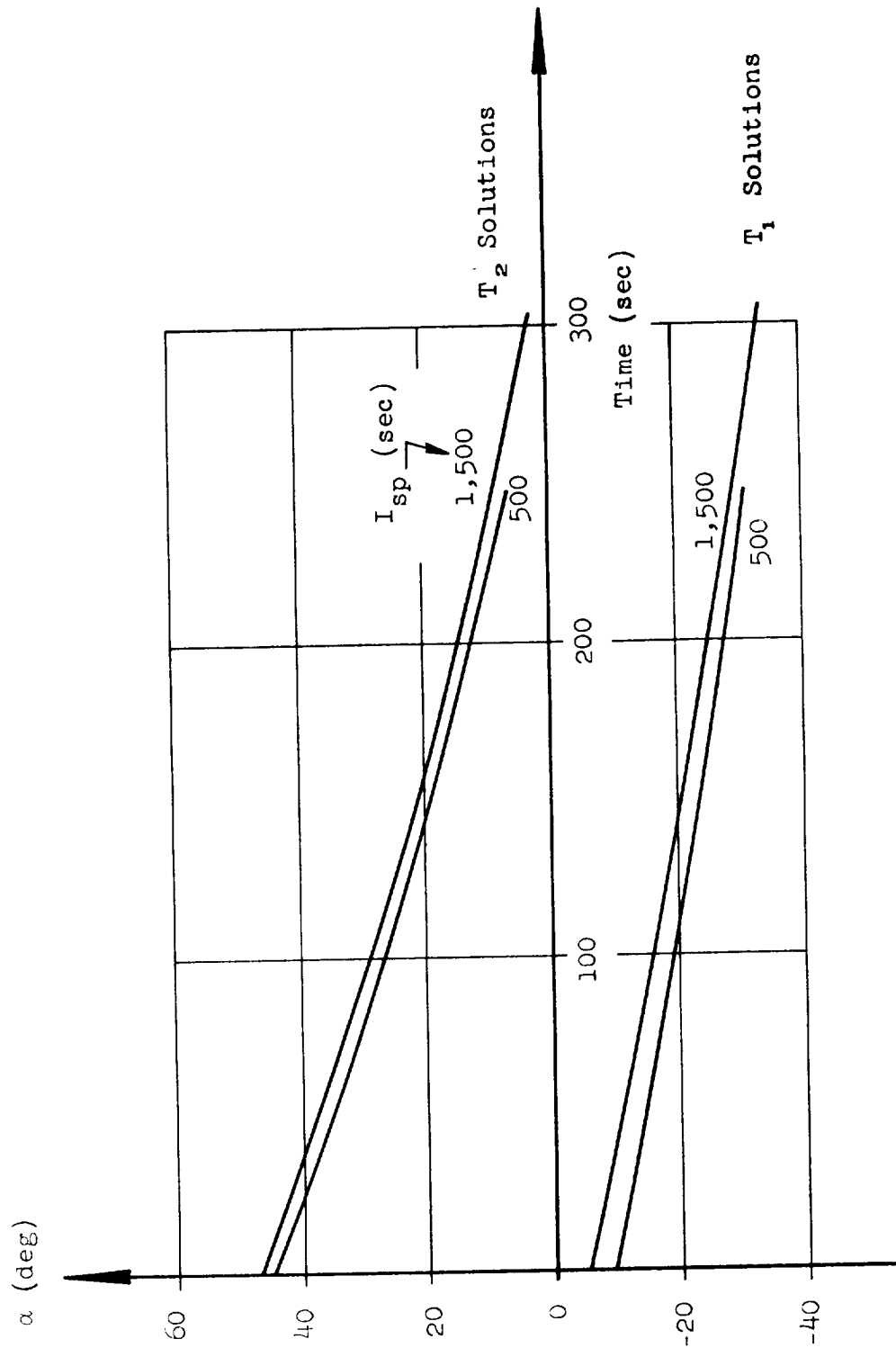
FOR $F/W_o = .5$; $y_o = 300$ km

FIG. 67.   OPTIMUM ANGLE OF ATTACK HISTORIES FOR PARAMETERS OF I_{sp}
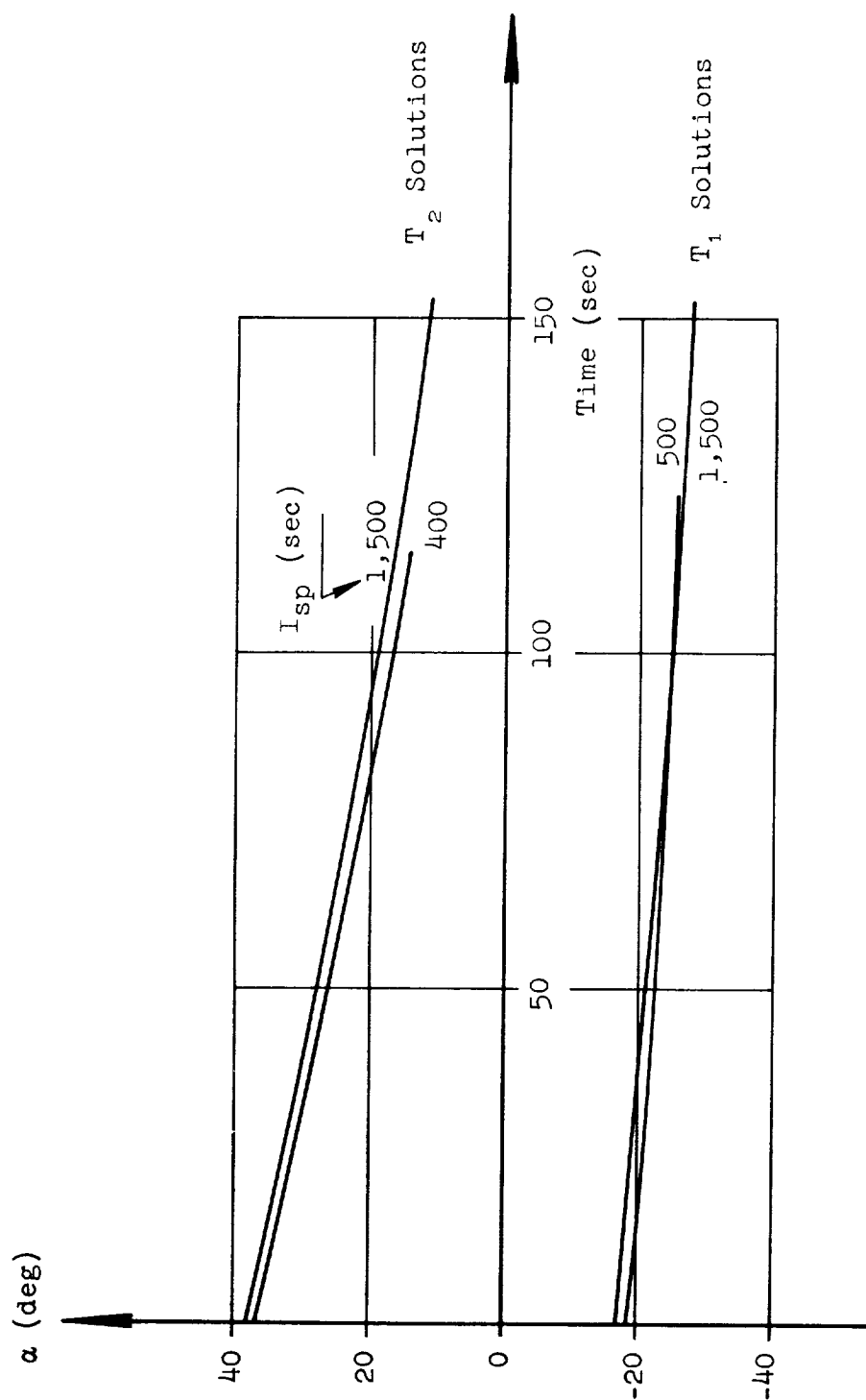
FOR $F/W_o = 1$; $y_o = 300$ km

FIG. 68. OPTIMUM ANGLE OF ATTACK HISTORIES FOR PARAMETERS OF I_{sp}

FOR $F/W_O = 2;$ $y_O = 300$ km

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A SURVEY OF THE INFLUENCE OF VARIATIONS
IN STAGE, CHARACTERISTICS ON OPTIMIZED TRAJECTORY SHAPING

PART II: TRANSFER FROM CIRCULAR ORBITS
INTO A SPACE FIXED ELLIPSE

By

Gary McDaniel

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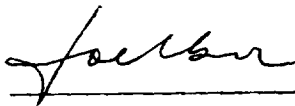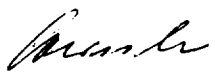ORIGINATOR   APPROVAL

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